



Moment Determinacy of Products of Non-identically Distributed Random Variables

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There is a long standing interest in studying products of random variables. In general, it is rare to find explicit closed-form expressions for the densities or the distributions of products of random variables with different distributions. It is, however, possible to study successfully the moment problem for products of independent random variables. Answers about the moment (in)determinacy can be found if requiring only information about the asymptotics of the moments or about the tails of the densities or of their distributions. We show first that there are intrinsic relationships among different conditions, old and recent, which lead to some general statements in both the Stieltjes and the Hamburger moment problems. Then we describe checkable conditions and prove new results about the moment (in)determinacy for products of independent and non-identically distributed random variables. We treat all three cases when the random variables are nonnegative (Stieltjes case), when they take values in the whole real line (Hamburger case), and the mixed case. As an illustration we characterize the moment determinacy of products of random variables whose distributions are generalized gamma or double generalized gamma all with distinct shape parameters. Among other corollaries, the product of two independent random variables, one exponential and one inverse Gaussian, is moment determinate, while the product is moment indeterminate for the cases: one exponential and one normal, one chi-square and one normal, and one inverse Gaussian and one normal.

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