Prediction of Survival Times of Censored Items in a Simple Step-Stress Model with Progressive Type II Censoring

Indrani Basak*
Penn State Altoona, Altoona, USA - i8b@psu.edu

N. Balakrishnan
McMaster University, Hamilton, Canada - bala@univmail.cis.mcmaster.ca

Abstract

In this article, we consider the problem of predicting survival times of units from the Exponential distribution which are censored under a simple step-stress model. Progressive Type II censoring are considered. Two kinds of predictors - the maximum likelihood predictors (MLP) and the conditional median predictors (CMP) are derived. Some numerical examples are presented to illustrate the prediction methods discussed here. Using simulation studies, prediction intervals are generated for these examples. Then we compare the MLP and the CMP with respect to mean squared prediction error (MSPE) and the prediction interval.

Keywords: Conditional median predictor; Maximum likelihood predictor; Mean squared prediction error; Prediction Interval.

1. Introduction

In recent days, the products which are tested in industrial experiments are extremely reliable with large mean survival times under normal operating conditions. In order to reduce cost and time of the experimenter, the complete survival data are not observed by the experimenter for a portion of the items under study. Those items can be censored intentionally. In conventional industrial life-testing experiment, even with the efficient censoring schemes, it is almost impossible to obtain adequate information about survival time distribution because of the extreme reliability of the products. Accelerated life tests allow the experimenter to expose the experimental units in industrial experiments under extreme stress factors in order to obtain information on the survival time distribution more rapidly than under normal operating conditions. A special class of accelerated life tests are step-stress tests that allow the experimenter to change the stress levels at pre-fixed times during the life-testing experiment. In this article, we consider a simple step-stress testing with only two stress levels which has been extensively studied in the literature. For the step-stress model, statistical inferential methods for survival time distributions have already been suggested in the literature [see for example, Balakrishnan et. al.(2007), Balakrishnan, N. & Xie, Q. (2007)]. But the prediction of actual survival times (which are censored) has captured very little attention. Prediction of unobserved or censored observations is an interesting topic, especially in the viewpoint of actuarial, medical and engineering sciences. Developing statistical prediction theory and procedures for step-stress models under progressive Type II censoring schemes based on exponential distribution is the purpose of this article. We would be using a frequently used predictor which is called the maximum likelihood predictor (MLP). Conditional median predictor (CMP) is another possible predictor.

2. Prediction for Simple Step-Stress Model Under Progressive Type II Censoring

Suppose a sample of n experimental units are placed on a simple step-stress life test at an initial stress level of s_1 and the stress level is changed to s_2 at a pre-fixed time τ. Then, the progressive Type-II censoring is implemented in this experimental setting in the following manner. At the stress-level s_1 and at the time of the first failure, $R_{1,1}$ of the $n-1$ surviving units are randomly removed from the experiment. At the time of the second failure, $R_{1,2}$ of the $n-2-R_{1,1}$ surviving units are randomly removed from the experiment, and similarly the test continues before time τ. Let N_1 be the number of units that fail at stress level s_1 and

$R_1 = \sum_{i=1}^{N_1} R_{1,i}$ be the total number of the censored units at stress level s_1. Then, after time τ (at stress level s_2), at the time of the (N_1+1)-th failure, R_{2,N_1+1} of the $n - N_1 - R_1 - 1$ surviving units are randomly removed from the experiment. At the time of the (N_1+2)-th failure, R_{2,N_1+2} of the $n - N_1 - R_1 - R_{2,N_1+1} - 2$ surviving units are randomly removed from the experiment.

Similarly the test continues at the stress level s_2. Let $R_2 = \sum_{i=N_1+1}^{N} R_{2,i}$ be the total number of the censored units at stress level s_2 for a fixed value of N, the total number of observations. Then let $N_2 = N - N_1$ be the number of units that fail at stress level s_2. With $N, R_{1,i}(i = 1, 2, \cdots, N_1)$ and $R_{2,i}(i = N_1+1, \cdots, N-1)$ fixed in advance, the test continues until the N-th failure at which time all the remaining $n - N_1 - N_2 - R_1 - \sum_{i=N_1+1}^{N-1} R_{2,i}$ surviving units are removed. Note that $n = N_1 + N_2 + R_1 + R_2$. If $R_{1,1} = \cdots = R_{1,N_1} = 0$ and $R_{2,N_1+1} = \cdots = R_{2,N} = 0$ then $n = N$ which corresponds to the complete sample situation. If $R_{1,1} = \cdots = R_{1,N_1} = 0$, $R_{2,N_1+1} = \cdots = R_{2,N-1} = 0$ and $R_{2,N} = n - N$ then it corresponds to the conventional Type-II right censoring scheme. Note that the life-testing experiment is terminated when the N-th failure occurs. With these notations, we will observe the following progressively censored data:

$$\boldsymbol{t} = \{t_1 < \cdots < t_{N_1} < \tau \le t_{N_1+1} < \cdots < t_N\}. \tag{1}$$

$\boldsymbol{t}$ is the observed values of the variable $\boldsymbol{T} = (T_1, \cdots, T_{N_1}, T_{N_1+1}, \cdots, T_N)$ denoting the N Type-II progressively right censored order statistics from a population with pdf $g(t) = g(t;\boldsymbol{\theta})$ where $(t;\boldsymbol{\theta}) \in \boldsymbol{D} = (R^+)^r \times \Omega$. Here, $\boldsymbol{\theta} = (\theta_1, \theta_2)$ and Ω is a 2-dimensional parameter vector.

2.1 Maximum Likelihood Predictor

Let $(R_{1,1}, \cdots, R_{1,N_1}; R_{2,N_1+1}, \cdots, R_{2,N})$ be denoted by $(R_1, \cdots, R_N)$. We now consider the maximum likelihood prediction of $T_{j:R_i}$, $(j = 1, 2, \cdots, R_i;\ i = 1, 2, \cdots, N)$, having observed $\boldsymbol{T}$. Here, $\boldsymbol{t} = (t_1, \cdots, t_{N_1}, t_{N_1+1}, \cdots, t_N)$ and $t_{j:R_i} = t$ denote the observed value of $\boldsymbol{T}$ and the unobserved value of $T_{j:R_i}$, respectively. Using the fact that the conditional density of $T_{j:R_i}$ is the same as the density of the j-th order statistic out of R_i units from the density $g(t)/(1 - G(t_i))$, $t \ge T_i$, the predictive likelihood function (PLF) of $T_{j:R_i}$ and $\boldsymbol{\theta}$ is given by

$$\begin{aligned} L &= L(t, \boldsymbol{\theta}; \boldsymbol{t}) \\ &= g_{T_{j:R_i}|\boldsymbol{t}}(t|\boldsymbol{t}, \boldsymbol{\theta}) \cdot g(\boldsymbol{t}, \boldsymbol{\theta}) \\ &= g_{T_{j:R_i}|T_i}(t|t_i, \boldsymbol{\theta}) \cdot g(\boldsymbol{t}, \boldsymbol{\theta}). \end{aligned} \tag{2}$$

L in (2) is then given by

$$L = \begin{cases} c_1 \prod_{l=1}^{N_1} g_1(t_l) \prod_{l=1, l\neq i}^{N_1} [1 - G_1(t_l)]^{R_l} [G_1(t) - G_1(t_i)]^{j-1} \\ g_1(t)[1 - G_1(t)]^{R_i - j} \prod_{l=N_1+1}^{N} g_2(t_l) \prod_{l=N_1+1}^{N} [1 - G_2(t_l)]^{R_l} & \text{if } 1 \le N_1 \le N-1 \\ & \ \&\ \ i = 1, \cdots, N_1, \\ \\ c_2 \prod_{l=1}^{N_1} g_1(t_l) \prod_{l=1}^{N_1} [1 - G_1(t_l)]^{R_l} \prod_{l=N_1+1}^{N} g_2(t_l) \\ \prod_{l=N_1+1, l\neq i}^{N} [1 - G_2(t_l)]^{R_l} [G_2(t) - G_2(t_i)]^{j-1} \\ g_2(t)[1 - G_2(t)]^{R_i - j} & \text{if } 0 \le N_1 \le N-1 \\ & \ \&\ \ i = N_1+1, \cdots, N. \end{cases} \tag{3}$$

Here c_1 and c_3 denote constant factors. If $T^L_{j:R_i} = u(\boldsymbol{T})$ and $\boldsymbol{\theta}^{**} = v(\boldsymbol{T})$ are statistics for which

$$L(u(\boldsymbol{T}), v(\boldsymbol{T}); \boldsymbol{T}) = \sup_{T,\theta} L(T, \boldsymbol{\theta}; \boldsymbol{T}),$$

then $u(\boldsymbol{T})$ is said to be the MLP of $T_{j:R_i}$ and $v(\boldsymbol{T})$ the predictive maximum likelihood estimator (PMLE) of $\boldsymbol{\theta} = (\theta_1, \theta_2)$. We will assume that the lifetimes will be exponentially distributed at all stress levels.

Case 1 : ($1 \leq N_1 \leq N-1$ and $i = 1, \cdots, N_1$)
The logarithm of the predictive likelihood function (log PLF) of $T_{j:R_i} = t$, corresponding to the PLF given by the first equation of (3), is given by

$$\begin{aligned} \log L &= -(N_1+1)\log\theta_1 - \frac{Q_1}{\theta_1} + (j-1)\log(1-e^{-\frac{t-t_i}{\theta_1}}) \\ &\quad -N_2\log\theta_2 - \frac{Q_2}{\theta_2} + \tau\left(\frac{1}{\theta_2} - \frac{1}{\theta_1}\right)\sum_{l=N_1+1}^{N}(R_l+1) \end{aligned} \tag{4}$$

in which $Q_1 = \sum_{l=1}^{N_1}(R_l+1)t_l + (R_i - j + 1)(t - t_i)$ and $Q_2 = \sum_{l=N_1+1}^{N}(R_l+1)t_l$.

The predictive likelihood equations (PLEs) are obtained by differentiating $\log L$ in (4) with respect to t and θ_1 and these are as follows:

$$\left.\begin{aligned} \frac{\partial \log L}{\partial t} &= -\frac{R_i-j+1}{\theta_1} + \frac{j-1}{\theta_1}\cdot\frac{e^{-\frac{t-t_i}{\theta_1}}}{1-e^{-\frac{t-t_i}{\theta_1}}} = 0, \\ \frac{\partial \log L}{\partial \theta_1} &= -\frac{N_1+1}{\theta_1} + \frac{Q_1 + \tau\sum_{l=N_1+1}^{N}(R_l+1)}{\theta_1^2} - \frac{j-1}{\theta_1^2}\cdot\frac{(t-t_i)e^{-\frac{t-t_i}{\theta_1}}}{1-e^{-\frac{t-t_i}{\theta_1}}}. \end{aligned}\right\} \tag{5}$$

The MLP $T^L_{j:R_i}$ of $T_{j:R_i}$ is given by

$$T^L_{j:R_i} = t_i + \theta_1^* \cdot D_1(j, R_i) \tag{6}$$

in which $D_1(j, R_i) = \log\left(\frac{R_i}{R_i-j+1}\right)$. The PMLE θ_1^* of θ_1 is then computed to be

$$\theta_1^* = \frac{\sum_{l=1}^{N_1}(R_l+1)t_l + \tau\sum_{l=N_1+1}^{N}(R_l+1)}{N_1+1}. \tag{7}$$

Case 2 : ($0 \leq N_1 \leq N-1$ and $i = N_1+1, \cdots, N$)
The logarithm of the predictive likelihood function (log PLF) of $T_{j:R_i} = t$, corresponding to the PLF given by the second equation of (3), is

$$\begin{aligned} \log L &= -N_1\log\theta_1 - \frac{M_1}{\theta_1} + (j-1)\log(1-e^{-\frac{t-t_i}{\theta_2}}) \\ &\quad -(N_2+1)\log\theta_2 - \frac{M_2}{\theta_2} + \tau(\frac{1}{\theta_2} - \frac{1}{\theta_1})\sum_{l=N_1+1}^{N}(R_l+1) \end{aligned} \tag{8}$$

in which $M_1 = \sum_{l=1}^{N_1}(R_l+1)t_l$ and $M_2 = \sum_{l=N_1+1}^{N}(R_l+1)t_l + (R_i - j + 1)(t - t_i)$.

The predictive likelihood equations (PLEs) are obtained by differentiating $\log L$ in (8) with respect to t and θ_2 and these are as follows:

$$\left.\begin{aligned} \frac{\partial \log L}{\partial t} &= -\frac{R_i-j+1}{\theta_2}+\frac{j-1}{\theta_2}\cdot\frac{e^{-\frac{t-t_i}{\theta_2}}}{1-e^{-\frac{t-t_i}{\theta_2}}}=0, \\ \frac{\partial \log L}{\partial \theta_2} &= -\frac{N_2+1}{\theta_2}+\frac{M_2-\tau\sum\limits_{l=N_1+1}^{N}(R_l+1)}{\theta_2^2}-\frac{j-1}{\theta_2^2}\cdot\frac{(t-t_i)e^{-\frac{t-t_i}{\theta_2}}}{1-e^{-\frac{t-t_i}{\theta_2}}}. \end{aligned}\right\} \tag{9}$$

The MLP $T^L_{j:R_i}$ of $T_{j:R_i}$ is given by

$$T^L_{j:R_i}=t_i+\theta_2^*\cdot D_1(j,R_i). \tag{10}$$

The PMLE θ_2^* of θ_2 is computed to be

$$\theta_2^*=\frac{\sum\limits_{l=N_1+1}^{N}(R_l+1)(t_l-\tau)}{N_2+1}. \tag{11}$$

2.2 Conditional Median Predictor

The median of the conditional distribution of $T_{j:R_i}$, given T_i, is called Conditional Median Predictor and will be referred to as CMP as mentioned earlier and will be denoted by $T^C_{j:R_i}$. The CMP $T^C_{j:R_i}$ of $T_{j:R_i}$ is such that $\int_{t_i}^{T^C_{j:R_i}} g(t|t_i)dt=\frac{1}{2}$ in which $g(t|t_i)$ is the conditional density of $T_{j:R_i}$, given t_i. Since $[T_{j:R_i}-T_i|T_i=t_i]\overset{d}{=}\theta\cdot Z_{j:R_i}$, the CMP $T^C_{j:R_i}$ of $T_{j:R_i}$ is given by

$$T^C_{j:R_i}=t_i+\theta\cdot M_{j:R_i} \tag{12}$$

where $Z_{j:R_i}$ is the j-th order statistic out of R_i units from Exp(1) and $\text{Med}\,[Z_{j:R_i}]=M_{j:R_i}$.

Case 1 : $(1\le N_1\le N-1$ **and** $i=1,\cdots,N_1)$ The CMP $T^C_{j:R_i}$ of $T_{j:R_i}$ is given by

$$T^C_{j:R_i}=t_i+\hat{\theta}_1\cdot M_{j:R_i} \tag{13}$$

where $\hat{\theta}_1$, the UMVUE of θ_1 is given by

$$\hat{\theta}_1=\frac{\sum\limits_{l=1}^{N_1}(R_l+1)t_l+\tau\sum\limits_{l=N_1+1}^{N}(R_l+1)}{N_1}. \tag{14}$$

Case 2 : $(0\le N_1\le N-1$ **and** $i=N_1+1,\cdots,N)$ The CMP $T^C_{j:R_i}$ of $T_{j:R_i}$ is given by

$$T^C_{j:R_i}=t_i+\hat{\theta}_2\cdot M_{j:R_i} \tag{15}$$

where $\hat{\theta}_2$, the UMVUE of θ_2, is given by

$$\hat{\theta}_2=\frac{\sum\limits_{l=N_1+1}^{N}(R_l+1)(t_l-\tau)}{N_2}. \tag{16}$$

3. Numerical Illustration

We generated a random sample of size $n = 40$ from the Exponential distribution under the step-stress setting with $\theta_1 = e^{3.0}, \theta_2 = e^{2.0}$ with $\tau = 15$ and is presented in Table 1. These data are then used to illustrate the prediction methods described in the preceding section.

Table 1: A random sample of size $n = 40$ with $\theta_1 = e^{3.0}, \theta_2 = e^{2.0}$ with $\tau = 15$

Parameters	Times-to-failure								
$\theta_1 = e^{3.0}$	0.22	1.16	1.45	1.58	2.92	3.70	4.30	6.20	7.23
	8.79	9.35	9.68	9.89	10.95	11.55	12.48	13.56	
$\theta_2 = e^{2.0}$	15.27	15.37	15.61	16.38	18.34	18.60	19.16	19.42	20.08
	21.00	21.06	21.96	22.29	24.42	24.68	24.82	25.54	26.35
	28.92	29.90	29.94	40.19	47.92				

Data in Table 1 were used to generate progressive Type II censored data under the step-stress setting using following schemes: At the first stress level with $\theta_1 = e^{3.0}$, we took $R_{1,1} = R_{1,2} = 0$ and at the time of the third failure, $R_{1,3} = c_1$ [for a pre-fixed c_1 value] of $n - 3$ surviving units are randomly censored from the experiment. Then, we took $R_{1,4} = R_{1,5} = R_{1,6} = 0$ and then at the time of the seventh failure$R_{1,7} = c_2$ [for a pre-fixed c_2 value] of $n - 7 - c_1$ surviving units are randomly censored from the experiment. Test continued before time $\tau = 15$ without any more censoring and total number of failures N_1 was observed at this first level of stress. Then, after time $\tau = 15$ and at the second stress level with $\theta_2 = e^{2.0}$, we took $R_{2,N_1+1} = \cdots = R_{2,N_1+4} = 0$ and then $R_{2,N_1+5} = c_3$ [for a pre-fixed c_3 value] of $n - N_1 - R_1 - 5$ surviving units are randomly censored from the experiment. Test continued until a pre-fixed number N of total failures occurred. At the time of N-th failure, all the remaining $n - N - R_1 - c_3$ units are censored where $R_1 = c_1 + c_2$.

Two progressive Type II censoring schemes were considered for the pre-fixed value of $N = 30$. In each scheme, we observed N_1 and then the total number of failures at the second level of stress $N_2 = N - N_1$ was observed. The first scheme has a faster rate of censoring in which we considered $c_1 = 3, c_2 = 3, c_3 = 2$ and so at the time of N-th failure, $n - N - R_1 - c_3 = 2$ values are censored. For this first scheme, we observed $N_1 = 13$ and so $N_2 = 17$. This scheme will be denoted by PCS-1. The second scheme has a delayed rate of censoring in which we considered $c_1 = 2, c_2 = 2, c_3 = 3$ and so at the time of N-th failure, $n - N - R_1 - c_3 = 3$ values are censored. For this second scheme, we observed $N_1 = 14$ and so $N_2 = 16$. This scheme will be denoted by PCS-2. So, summarizing these descriptions of PCS-1 and PCS-2, we have:

PCS-1: $N = 30(n = 40, N_1 = 13, N_2 = 17, R_{1,1} = R_{1,2} = 0, R_{1,3} = 3, R_{1,4} = R_{1,5} = R_{1,6} = 0, R_{1,7} = 3, R_{1,8} = \cdots = R_{1,N_1} = 0; R_{2,N_1+1} = \cdots = R_{2,N_1+4} = 0, R_{2,N_1+5} = 2, R_{2,N_1+6} = \cdots = R_{2,27} = 0, R_{2,28} = 2)$.

PCS-2: $N = 30(n = 40, N_1 = 14, N_2 = 16, R_{1,1} = R_{1,2} = 0, R_{1,3} = 2, R_{1,4} = R_{1,5} = R_{1,6} = 0, R_{1,7} = 2, R_{1,8} = \cdots = R_{1,N_1} = 0; R_{2,N_1+1} = \cdots = R_{2,N_1+4} = 0, R_{2,N_1+5} = 3, R_{2,N_1+6} = \cdots = R_{2,26} = 0, R_{2,27} = 3)$.

Another two progressive Type II censoring schemes censoring were considered for the pre-fixed value of $N = 33$. In each scheme, we observed N_1, the total number of failures at the first level of stress and then, $N_2 = N - N_1$, the total number of failures at the second level of stress was observed. The first scheme has a faster rate of censoring in which we considered $c_1 = 3, c_2 = 2, c_3 = 1$ and so at the time of N-th failure, $n - N - R_1 - c_3 = 1$ values are censored. For this first scheme, we observed $N_1 = 14$ and so $N_2 = 19$. This scheme will be denoted by PCS-3. The second scheme has a delayed rate of censoring in which we considered $c_1 = 1, c_2 = 1, c_3 = 2$ and so at the time of N-th failure, $n - N - R_1 - c_3 = 3$ values are censored. For this second scheme, we observed $N_1 = 16$ and so $N_2 = 17$. This scheme will be denoted by PCS-4. Again, summarizing these descriptions of PCS-3 and PCS-4, we have:

PCS-3: $N = 33(n = 40, N_1 = 14, N_2 = 19, R_{1,1} = R_{1,2} = 0, R_{1,3} = 3, R_{1,4} = R_{1,5} = R_{1,6} = 0, R_{1,7} = 2, R_{1,8} = \cdots = R_{1,N_1} = 0; R_{2,N_1+1} = \cdots = R_{2,N_1+4} = 0, R_{2,N_1+5} = 1, R_{2,N_1+6} = \cdots = R_{2,31} = 0, R_{2,32} = 1)$

and

PCS-4: $N = 33(n = 40, N_1 = 16, N_2 = 17, R_{1,1} = R_{1,2} = 0, R_{1,3} = 1, R_{1,4} = R_{1,5} = R_{1,6} = 0, R_{1,7} = 1, R_{1,8} = \cdots = R_{1,N_1} = 0; R_{2,N_1+1} = \cdots = R_{2,N_1+4} = 0, R_{2,N_1+5} = 2, R_{2,N_1+6} = \cdots = R_{2,29} = 0, R_{2,30} = 3)$.

We considered a second sample of larger sample size n=80 and repeated similar simulations with the following four progressive censoring schemes:

PCS-5: $N = 60(n = 80, N_1 = 36, N_2 = 24, R_{1,1} = R_{1,2} = 0, R_{1,3} = 6, R_{1,4} = R_{1,5} = R_{1,6} = 0, R_{1,7} = 6, R_{1,8} = \cdots = R_{1,N_1} = 0; R_{2,N_1+1} = \cdots = R_{2,N_1+4} = 0, R_{2,N_1+5} = 4, R_{2,N_1+6} = \cdots = R_{2,55} = 0, R_{2,56} = 4)$;

PCS-6: $N = 60(n = 80, N_1 = 37, N_2 = 23, R_{1,1} = R_{1,2} = 0, R_{1,3} = 4, R_{1,4} = R_{1,5} = R_{1,6} = 0, R_{1,7} = 4, R_{1,8} = \cdots = R_{1,N_1} = 0; R_{2,N_1+1} = \cdots = R_{2,N_1+4} = 0, R_{2,N_1+5} = 6, R_{2,N_1+6} = \cdots = R_{2,53} = 0, R_{2,54} = 6)$;

PCS-7: $N = 66(n = 80, N_1 = 37, N_2 = 29, R_{1,1} = R_{1,2} = 0, R_{1,3} = 6, R_{1,4} = R_{1,5} = R_{1,6} = 0, R_{1,7} = 4, R_{1,8} = \cdots = R_{1,N_1} = 0; R_{2,N_1+1} = \cdots = R_{2,N_1+4} = 0, R_{2,N_1+5} = 2, R_{2,N_1+6} = \cdots = R_{2,63} = 0, R_{64} = 2)$;

PCS-8: $N = 66(n = 80, N_1 = 39, N_2 = 27, R_{1,1} = R_{1,2} = 0, R_{1,3} = 2, R_{1,4} = R_{1,5} = R_{1,6} = 0, R_{1,7} = 2, R_{1,8} = \cdots = R_{1,N_1} = 0; R_{2,N_1+1} = \cdots = R_{2,N_1+4} = 0, R_{2,N_1+5} = 4, R_{2,N_1+6} = \cdots = R_{2,59} = 0, R_{2,60} = 6)$.

We computed the values of MLP and CMP of $T_{j:R_i}$. Then, we carried out a numerical study to compare the performances of these MLP and CMP in terms of their MSPEs. Moreover, using simulation studies, standard errors of these $T_{j:R_i}$ were generated and the prediction intervals were constructed for each of the predictors MLP and CMP in each situation.

For the sake of abbreviation, we are not reporting the simulation results.

5. Conclusions

In this article, we have derived the MLP and CMP for the survival times of units from the exponential distribution which are progressively Type II censored under a simple step-stress model. Bias and MSPE of these predictors are also derived. It is noted that these two predictors are quite easy to compute. We used simulation studies in order to illustrate and compare the methods developed in this article. It is found that the predicted values for CMP are generally closer to their actual values than the corresponding predicted values for MLP. Simulation studies show that the prediction method using CMP yield the closest prediction result particularly for larger sample size, larger number of uncensored observations and for delayed censoring scheme as long as the number of predicted observations are not very small. It is also observed that the bias for the predicted MLP values became smaller by increasing the sample size. The MLPs have smaller MSPEs than the CMPs. But the ratio of MSPEs of CMPs to the MSPEs of MLPs became closer to 1 as sample size increased.

References

Balakrishnan, N. (2009). A synthesis of exact inferential results for exponential step-stress models and associated optimal accelerated life-tests. *Metrika* **69**, pp. 351-396.

Balakrishnan, N. Kundu, D., Ng, H. K. T and Kannan, N. (2007). Point and interval estimation for a simple step-stress model with Type-II censoring, *Journal of Quality and Technology* **9**, pp. 35-47.

Balakrishnan, N. and Xie, Q. (2007). Exact inference for a simple step-stress model with Type-II hybrid censored data from the exponential distribution, *Journal of Statistical Planning and Inference* **137**, pp. 2543-2563.