Economic efficiency of the AOQL sampling plans for inspection by variables when the remainder of rejected lots is inspected

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Abstract

The average outgoing quality limit (AOQL) sampling plans minimizing the mean inspection cost per lot of process average quality when the remainder of rejected lots is inspected were originally designed for the inspection by attributes. The single sampling plans for the inspection by variables and for the inspection by variables and attributes (all items from the sample are inspected by variables, remainder of rejected lots is inspected by attributes) were then proposed. Under the same protection of consumer the AOQL plans for inspection by variables are in many situations more economical than the corresponding sampling plans for inspection by attributes. It is shown how a comparative efficiency of a sampling plan for inspection by variables and attributes can be assessed using a measure which has been proposed for this purpose and implemented in a contributed package for the R computing environment.

Keywords: acceptance sampling; cost of inspection.

1. Introduction

Under the assumption that each inspected item is classified as either good or defective (acceptance sampling by attributes), Dodge & Romig (1998) introduced sampling plans (n, c) which minimize the mean number of items inspected per lot of the process average quality, assuming that the remainder of the rejected lots is inspected

$$I_s = N - (N - n) \cdot L(\bar{p}; n, c) \tag{1}$$

under the condition

$$\max_{0<p<1} AOQ(p) = p_L. \tag{2}$$

The notation in equations (1) and (2) is as follows:
N is the number of items in the lot (the given parameter),
$\bar{p}$ is the process average fraction defective (the given parameter),
p_L is the average outgoing quality limit (the given parameter, denoted AOQL),
n is the number of items in the sample ($n < N$),
c is the acceptance number (the lot is rejected when the number of defective items in the sample is greater than c),
$L(p)$ is the operating characteristic (the probability of accepting a submitted lot with the fraction defective p).
The function AOQ is the *average outgoing quality,* $AOQ(p)$ is the mean fraction defective after inspection when the fraction defective before inspection was p. The average outgoing quality (where all defective items found are replaced by good ones) is approximately

$$AOQ(p) = \left(1 - \frac{n}{N}\right) \cdot p \cdot L(p; n, c). \tag{3}$$

Therefore condition (2) can be rewritten as

$$\max_{0<p<1} \left(1 - \frac{n}{N}\right) \cdot p \cdot L(p; n, c) = p_L. \tag{4}$$

Condition (2) protects the consumer against having an average outgoing quality higher than p_L (the chosen value), regardless of what the fraction defective p is before inspection.
The AOQL plans for the inspection by variables and attributes (all items in the sample are inspected by variables, the remainder in rejected lot is inspected by attributes) have been introduced in Klufa (1997), using an approximate calculation of the plans. The new parameter, the ratio of the cost of inspecting an item by variables to the cost of inspecting an item by attributes c_m has been introduced. Exact plans, using non-central t distribution in calculation of the operating characteristic, have been reported in Klufa (2008). A measure for an assessment of a comparative efficiency of a sampling plan for inspection by variables and attributes has been proposed in Kasprikova & Klufa (2015). This paper shows how this measure can be used to help in deciding if inspection by variables should be considered instead of inspection by attributes and shows how the economic characteristics of the exact AOQL plans for the inspection by variables and attributes and can be calculated and how the economic efficiency of the sampling plan can be assessed.

2. Economic characteristics of AOQL plans for the inspection by variables and attributes

Example. There is a lot with $N = 8000$ items considered in the acceptance procedure. The average outgoing quality limit is given to be $p_L = 0.01$. It is known that the average process quality is $\bar{p} = 0.003$. A cost of inspecting an item by variables is 50% higher than the cost of inspecting the item by attributes, so the parameter c_m equals 1.5. We are to find the optimal AOQL acceptance sampling plan for sampling inspection by variables when the remainder of rejected lots is inspected by attributes.
The LTPDvar package (Kasprikova, 2012) for R software (R Core Team, 2013) can be used for the calculation of the sampling plan. The resulting plan is $n = 55$, $k = 2.076524$.
For the values of the input parameters given in our problem, there is plan (135, 2) for the acceptance sampling by attributes in Dodge & Romig (1998). Let us compare the plans ($n = 55$, $k = 2.076524$) and ($n = 135$, $c = 2$) with regard to the economic efficiency.
For the comparison of the AOQL sampling plans for the inspection by variables and attributes and the corresponding Dodge-Romig AOQL sampling plans for the inspection by attributes with regard to the economic point of view we will use parameter e, defined as

$$e = \frac{I_{ms}}{I_s} \cdot 100 \tag{5}$$

where $I_{ms} = n \cdot c_m + (N - n) \cdot (1 - L(\bar{p}; n; k))$.
Since

$$e = 54.37,$$

it can be expected that approximately **46% savings in the inspection cost** can be made using the AOQL plan for inspection by variables and attributes (55, 2.076524), in place of the corresponding Dodge-Romig plan $(135, 2)$.

Economical parameter e defined by (5) is a function of four variables p_L, N, $\bar{p}$ and c_m, i. e.

$$e = e(p_L, N, \bar{p}, c_m).$$

Let p_L, N, $\bar{p}$ be given parameters (for given p_L, N, $\bar{p}$ we shall write $e_{p_L,N,\bar{p}}(c_m)$ instead of $e = e(p_L, N, \bar{p}, c_m)$). Function $e_{p_L,N,\bar{p}}(c_m)$ (i.e. e for given parameters p_L, N, $\bar{p}$) is a function of one variable c_m.

Remark. From the results of numerical investigations it follows that the function $e(p_L, N, \bar{p})$ (i.e. e for given parameters p_L, N, $\bar{p}$) is increasing function of c_m (see also Figure 1).

Now let us calculate the value of the c_m^{BE} parameter, which has been introduced in Kasprikova & Klufa (2015). We shall recall the definition of the c_m^{BE} parameter first.

Definition. Let p_L, N, $\bar{p}$ be given parameters. Let us define c_m^{BE} as the value of c_m for which $e = 100$.

According to this definition, c_m^{BE} is such value of c_m for which the mean inspection cost per lot of process average quality for the inspection by variables and attributes is equal to the mean inspection cost per lot of process average quality for the inspection by attributes. If c_m is an estimate based on real cost calculation

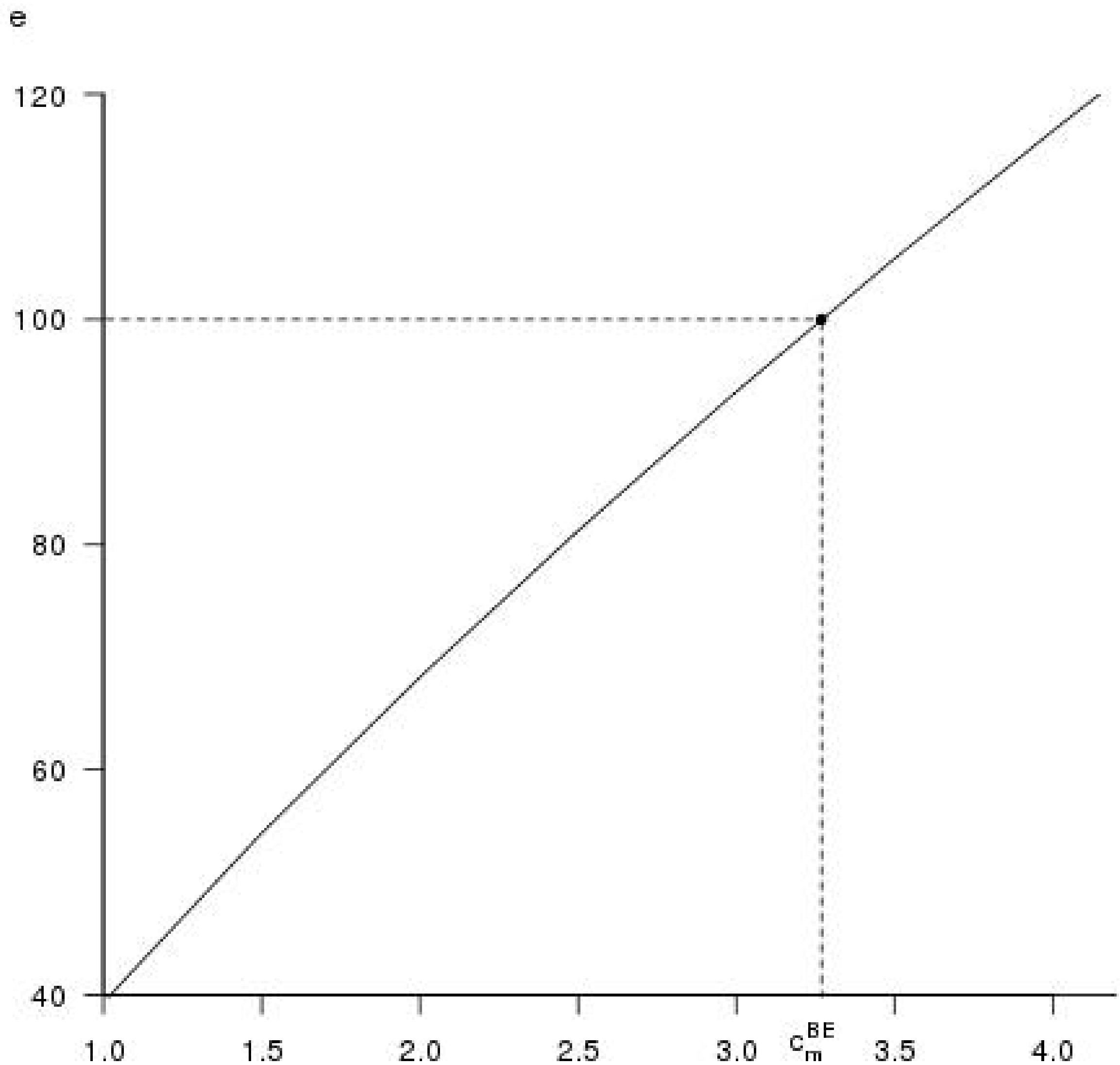

Figure 1: Graph of function $e = e(c_m)$ for $p_L = 0.01$, $N = 8000$, $\bar{p} = 0.003$

and $c_m < c_m^{BE}$, then the AOQL plans for inspection by variables and attributes are more economical than the corresponding Dodge-Romig AOQL plans. If $c_m > c_m^{BE}$, then the Dodge-Romig AOQL plans for inspection by attributes are more economical than the corresponding AOQL plans for the inspection by variables and attributes.
If the value of c_m parameter is not known in some situation in practice, then c_m^{BE} (the break-even value of c_m parameter) may be calculated to provide some guidance in deciding if inspection by variables and attributes is worth considering. If c_m^{BE} is high, then using inspection by variables and attributes may be efficient (and one should try to estimate c_m to make some more precise evaluation), on the other hand if c_m^{BE} is near 1, then the inspection by variables and attributes can not be supposed to bring significant advantage over the inspection by attributes. The calculation of c_m^{BE} value is implemented in the LTPDvar package (Kasprikova, 2012) using the `cmBE` function.

The value of the c_m^{BE} parameter in the example situation discussed above is 3.2698. It means that if the value of c_m parameter is below 3.2698, then there are savings in inspection cost to be expected if the inspection by variables and attributes is performed in place of the inspection by attributes. So for $c_m = 1.5$ the inspection by variables and attributes is supposed to be more economical than the inspection by attributes.

Now let us consider another situation.
Example. There is a lot with $N = 3500$ items considered in the acceptance procedure. The average outgoing quality limit is given to be $p_L = 0.015$. It is known that the average process quality is $\bar{p} = 0.01$. A cost of inspecting an item by variables is 70% higher than the cost of inspecting the item by attributes, so the parameter c_m equals 1.7. We are to find the optimal AOQL acceptance sampling plan for sampling inspection by variables when the remainder of rejected lots is inspected by attributes.
The LTPDvar package (Kasprikova, 2012) for R software (R Core Team, 2013) can be used for the calculation of the sampling plan. The resulting plan is $n = 74$, $k = 1.9208$.
For the values of the input parameters given in our problem, there is plan (165, 4) for the acceptance sampling by attributes in Dodge & Romig (1998). Let us compare the plans ($n = 74$, $k = 1.9208$) and ($n = 165$, $c = 4$) with regard to the economic efficiency. We shall use the e parameter again.
Since

$$e = 79.67,$$

it can be expected that approximately **20% savings in the inspection cost** can be made using the AOQL plan for inspection by variables and attributes (74, 1.9208), in place of the corresponding Dodge-Romig plan $(165, 4)$.
The value of the c_m^{BE} parameter is just 2.4101, which means that if the value of c_m parameter is below 2.4101, then there are savings in inspection cost to be expected if the inspection by variables and attributes is performed in place of the inspection by attributes. So for $c_m = 1.7$ the inspection by variables and attributes is supposed to be more economical than the inspection by attributes.

3. Conclusions

It was shown that the AOQL acceptance sampling plans for the inspection by variables and for the inspection by variables and attributes may bring considerable savings in the mean inspection cost per lot of process average quality in comparison with the corresponding Dodge-Romig acceptance sampling plans for the inspection by attributes and the extent of savings in the inspection cost is given by the input parameters values such as the ratio of the cost of inspection of one item by variables to the cost of inspection of the item by attributes c_m. The economic efficiency of the sampling plan for the inspection by variables and attributes in comparison with the plan for the inspection by attributes may be assessed using the break even value of c_m parameter.

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