Unified Hypothesis Testing for Bayesian and Frequentist Approaches

Ivair Silva
Department of Statistic, Federal University of Ouro Preto, Ouro Preto, MG, Brasil
Department of Population Medicine, Harvard Medical School and Harvard Pilgrim Healph Care Institute, Boston, MA, USA - ivairest@gmail.com

Hypothesis testing plays a fundamental role in the modern practice of statistical analysis. In statistical inference, a hypothesis is a statement about a population parameter $\theta \in \Re$. The objective of a statistical hypothesis test is to decide which of two hypotheses is true. The general format of the two hypotheses is $H_0 : \theta \in \Theta_0$ and $H_1 : \theta \in \Theta_1$, where Θ_i, $i = 0, 1$, is a subset of the parameter space, Θ, $\Theta_0 \cap \Theta_1 = \emptyset$, and they not necessarily form a partition of Θ. H_0 and H_1 are called the null hypothesis and the alternative hypothesis, respectively. The decision of taking H_i as true is based on a random sample, $X_1, \cdots, X_n$, from the population. In practice, an arbitrary real-valued function of the sample, a 'measure of evidence in favor of H_i', $i = 0, 1$, is the statistic used to define the decision rule. Small observed values for the measure of evidence in favor of H_i suggest rejection of H_i. The reasoning used to construct an evidence measure is the main point of divergence between the Bayesian and the frequentist approaches. In the frequentist approach, the well-known evidence measure is the p-value, and the focus is in the control of the Type I error probability. The Bayesian approach consists on using the observed sample from the population to update the analyst's uncertainty, apart the data, about the plausibility of H_i. With the Bayes rule, the analyst can then obtain the so called posterior uncertainty (posterior distribution) about the veracity of each hypothesis. The performance measure usually focused is the expected loss incurred by the possible wrong decisions. Currently, the efforts to accommodate both approaches under the same decision rule are not applicable for the general case of any hypothesis testing problem. This paper offers an unified approach that enables to place Bayesian and frequentist tests under the same decision rule. Additionally, a theorem is provided showing that Bayesian and frequentist approaches can be made always equivalent. The post-market vaccine safety surveillance problem is used to illustrate how to construct a test based on the proposed unified approach.

Keywords: Decision set; evidence measures; Bayes Factor; p-value; performance measures.