The Asymptotic Behavior of Likelihood Estimators for the Dispersion Parameter of the Negative Binomial Distribution

Neville Weber*
University of Sydney, Sydney, Australia
neville.weber@sydney.edu.au

Katherine Courtney
University of Sydney, Sydney, Australia

The random variable X has a negative binomial distribution, NBD(μ, α), if $P(X = x) = \Gamma(x+\alpha)/[\Gamma(\alpha)\ x!] \times p^{\alpha}(1-p)^{x}$, for $x = 0, 1, \ldots$, where $\mu = EX = \alpha(1-p)/p$, $\alpha > 0$ and $0 < p < 1$. There is general agreement that the sample mean is the best estimator for the parameter μ. Several alternative estimators have been proposed for the dispersion parameter, α, the simplest being the method of moments estimator, which is defined provided the sample variance is greater than the sample mean. In simulation studies the method of moments estimator, the conditional likelihood estimator (CLE) and the maximum likelihood estimator (MLE) have been found to be the most reliable estimators, but all three exhibit erratic behavior for particular parameter combinations. We develop asymptotic results to confirm that the CLE and MLE exhibit the same asymptotic behavior as the simple moment estimator. These results help to explain the observed heavy tailed behavior and provide an alternative perspective on when the various estimators are defined.

Keywords: Negative binomial distribution; maximum likelihood estimator; conditional likelihood estimator; weak limit theorem.