Multivariate Log-CFUSN Distribution: Bayesian inference and properties

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Abstract

We introduce the multivariate log canonical fundamental skew-normal (log-CFUSN) and discuss some of its properties, such as marginal distributions and Shannon entropy. This class of log-skewed distributions include the log-normal and multivariate log-skew normal families as particular cases. We discuss some issues related to Bayesian inference in the log-CFUSN family. This proposed family is used to analyze the US national monthly precipitation data. We conclude that a high dimensional skewing function lead to a better model fit.
Keywords: Entropy; MCMC; marginalization; skewness.

1. Introduction

The construction of new parametric distributions has received considerable attention in recent years. This growing interest is motivated by datasets that often present strong skewness, heavy tails, bimodality and some other characteristics that are not well fitted by the usual distributions. The seminal paper by Azzalini (1985), which introduced the skew-normal (SN) family of distributions, is one of the main references in this topic and has inspired many other works. Several classes of SN distributions were defined in the literature after that.

The fundamental SN (FUSN) class of distributions defined by Arellano-Valle and Genton (2005) is a generalization of the SN class. A vector Z^* has an n-variate canonical fundamental skew-normal (CFUSN) distribution with an $n \times m$ skewness matrix $\mathbf{\Delta}$, which will be denoted by $Z^* \sim CFUSN_{n,m}(\mathbf{\Delta})$, if its density is given by

$$f_{\mathbf{Z}^*}(\mathbf{z}) = 2^m \phi_n(\mathbf{z}) \Phi_m(\mathbf{\Delta}'\mathbf{z} | \mathbf{I}_m - \mathbf{\Delta}'\mathbf{\Delta}), \quad \mathbf{z} \in \mathbf{R}^n, \tag{1}$$

where $\mathbf{\Delta}$ is such that $||\mathbf{\Delta a}|| < 1$, for all unitary vectors $\mathbf{a} \in \mathbf{R}^m$, and $||\cdot||$ denotes euclidean norm. Along this paper, we denote by $\phi_n(\mathbf{y} \mid \boldsymbol{\mu}, \mathbf{\Sigma})$ the p.d.f. associated with the multivariate $N_n(\boldsymbol{\mu}, \mathbf{\Sigma})$ distribution, and by $\Phi_n(\mathbf{y} \mid \boldsymbol{\mu}, \mathbf{\Sigma})$ the corresponding cumulative distribution function (c.d.f.). If $\boldsymbol{\mu} = \mathbf{0}$ (respectively $\boldsymbol{\mu} = \mathbf{0}$ and $\mathbf{\Sigma} = \mathbf{I}_n$) these functions will be denoted by $\phi_n(\mathbf{y} \mid \mathbf{\Sigma})$ and $\Phi_n(\mathbf{y} \mid \mathbf{\Sigma})$ (respectively $\phi_n(\mathbf{y})$ and $\Phi_n(\mathbf{y})$). For simplicity, $\phi(y)$ and $\Phi(y)$ will be used in the univariate case.

In limit cases, the CFUSN distribution concentrate its probability mass in positive (or negative) values. Because of this, skewed distributions with real support has been frequently considered to model data with positive support, such as income, precipitation, pollutants concentration and so on. However, the limit distributions are not flexible enough to accommodate the diversity of shapes in positive (negative) data. In the multivariate context, distributions with positive support are usually intractable. Marchenko and Genton (2010) built a tractable family known as the multivariate log-skew elliptical family of distributions as follows. Consider the transformation $\exp(\mathbf{X}) = (\exp(X_1), \ldots, \exp(X_n))$, where $\mathbf{X} \sim SEl_n(\mu, \mathbf{\Sigma}, \alpha, g^{(n+1)})$. Then, $\mathbf{X}$ has log-skew elliptical distribution denoted by $\mathbf{X} \sim LSEl_n(\mu, \mathbf{\Sigma}, \alpha, g^{(n+1)})$ with pdf

$$f_{LSEl_n}(\mathbf{x}) = 2 \left(\prod_{i=1}^{n} x_i \right)^{-1} f_n(\ln(\mathbf{x}); \mu, \mathbf{\Sigma}, g^{(n)}) F(\alpha' \omega^{-\mathbf{1}}(\ln(\mathbf{x}) - \mu); g_{Q_{\mathbf{x}}^{\mu},\mathbf{\Sigma}}), \ \mathbf{x} > 0, \tag{2}$$

where $g^{(n)}(u), u \geq 0$ is a generating function defining an n-dimensional spherical density, a location column vector $\mu \in \mathbf{R}^n$, and an $n \times n$ positive definite dispersion matrix $\mathbf{\Sigma}$, $\alpha \in \mathbf{R}^n$ is a shape parameter, $\omega = diag(\mathbf{\Sigma})^{1/2}$, $f_n(\mathbf{x}; \mu, \mathbf{\Sigma}, g^{(n)})$ is the pdf of an n-dimensional random vector of $El_n(\mu, \mathbf{\Sigma}, g^{(n)})$ and $F(u; g_{Q_{\mathbf{x}}^{\mu},\mathbf{\Sigma}})$ is the cdf of the $El_1(0, 1, g_{Q_{\mathbf{x}}^{\mu},\mathbf{\Sigma}})$ with generating function $g_{Q_{\mathbf{x}}^{\mu},\mathbf{\Sigma}}(u) = g^{(n+1)}(u + Q_{\mathbf{x}}^{\mu,\mathbf{\Sigma}})/g^{(n)}(Q_{\mathbf{x}}^{\mu,\Sigma})$.

In this paper, we introduce the multivariate log-CFUSN of distributions and studied some of its properties. Such classes of distributions have as subclasses the multivariate log-skew-normal family introduced by Marchenko and Genton (2010), the log-SN family and the family of distributions given in (3). Shannon entropy the CFUSN and log-CFUSN families are also obtained. We also discuss some issues related to Bayesian inference in the log-CFUSN family. To illustrate its use we analyze the USA monthly precipitation data recorded from 1895 to 2007, that is available at the National Climatic Data Center (NCDC).
Our main motivation to introduce new classes of multivariate log-skewed distribution are some results that recently appeared in a paper by Santos et al.(2013). That paper focused on the parameter interpretation in the mixed logistic regression models. By considering the random effects, the odds ratio to compare two individuals in two different clusters becomes a random variable (OR) that depends on the random effects related to the two clusters under comparison. Because of this, the parameters are interpreted using the median of the odds ratio distribution in order to quantify appropriately the heterogeneity among the different clusters. If the random effects are independent and identically distributed (iid) with $SN(\xi,\sigma^2,\lambda)$ then Santos et al.(2013) prove that the odds ratio has distribution with pdf given by

$$f_{OR|\boldsymbol{\beta},\boldsymbol{\theta},\mathbf{x}}(r) = \frac{4}{r}\phi(\ln r|\kappa_{12}, 2\sigma^2) \times \Phi_2\left(\frac{\delta \ln r}{2\sigma}\boldsymbol{\epsilon}|\frac{\delta\kappa_{12}}{2\sigma}\boldsymbol{\epsilon}, \mathbf{I}_2 - \frac{\delta^2}{2}\boldsymbol{\epsilon}\boldsymbol{\epsilon}'\right), \quad r \in \mathbf{R}_+, \tag{3}$$

where $\kappa_{12} = (\mathbf{x}'_{i_1j_1} - \mathbf{x}'_{i_2j_2})\boldsymbol{\beta}$, $\boldsymbol{\epsilon} = (1,-1)'$ and $\delta = \lambda(1+\lambda^2)^{-0.5}$. The univariate log-skewed distribution in (3) does not belong to the class of distributions defined by Marchenko and Genton (2010). Moreover, only its median was obtained but no other property of it was studied.

2. The log-CFUSN family

Let $\mathbf{Z}^* = (Z_1^*, \ldots, Z_n^*)'$ be a column random vector of order n and consider the transformations $\exp \mathbf{Z}^* = (\exp(Z_1^*), \ldots, \exp(Z_n^*))'$ and $\ln \mathbf{Z}^* = (\ln Z_1^*, \ldots, \ln Z_n^*)'$. Let $\mathbf{Z}^*$ and $\mathbf{Y}$ be $n \times 1$ random vectors such that $\mathbf{Z}^* = \ln \mathbf{Y}$. We say that $\mathbf{Y}$ has a log-canonical-fundamental-skew-normal distribution with $n \times m$ skewness matrix $\boldsymbol{\Delta}$ denoted by $\mathbf{Y} \sim LCFUSN_{n,m}(\boldsymbol{\Delta})$, if $\mathbf{Z}^* \sim CFUSN_{n,m}(\boldsymbol{\Delta})$. The pdf of $\mathbf{Y}$ is

$$f_{\mathbf{Y}}(\mathbf{y}) = 2^m \left(\prod_{i=1}^n y_i\right)^{-1} \phi_n(\ln \mathbf{y})\Phi_m(\boldsymbol{\Delta}' \ln \mathbf{y}|\mathbf{I}_m - \boldsymbol{\Delta}'\boldsymbol{\Delta}), \quad \mathbf{y} \in \mathbf{R}^{n^+}, \tag{4}$$

where $\boldsymbol{\Delta}$ is an $n \times m$ matrix such that $||\boldsymbol{\Delta}\mathbf{a}|| < 1$, for all unity vectors $\mathbf{a} \in \mathbf{R}^m$.
This distribution generalizes the multivariate log-SN distribution defined by Marchenko and Genton (2010) by assuming a m-variate skewing function, which is obtained if, in (4) we take $m = 1$ and assume $\alpha = (\mathbf{I}_m - \boldsymbol{\Delta}'\boldsymbol{\Delta})^{-\frac{1}{2}}\boldsymbol{\Delta}'$. If $\boldsymbol{\Delta}$ is a matrix with all entries equal to zero we have the multivariate log-normal distribution. Another reason to study this distribution comes from results summarized in the introduction. As it can be noticed, the distribution for the odds ratio given in (3) also belongs to the log-CFUSN family of distributions whenever the individuals under comparison have the same characteristics, that is, equal vector of covariates ($\mathbf{x}^t_{i_1j_1} = \mathbf{x}^t_{i_2j_2}$), and the scale parameter for the distribution of the random effects is $\sigma^2 = 1$. In that case, $OR \sim LCFUSN_{1,2}(\boldsymbol{\Delta})$ where $\boldsymbol{\Delta} = \delta\boldsymbol{\epsilon}$.
Similar to what is observed for the CFUSN family of distributions, the log-CFUSN is closed under marginalization. Let $\mathbf{Y} \sim LCFUSN_{n,m}(\boldsymbol{\Delta})$ and consider the partitions $\mathbf{Y} = (\mathbf{Y}_1', \mathbf{Y}_2')'$ and $\boldsymbol{\Delta} = (\boldsymbol{\Delta}_1', \boldsymbol{\Delta}_2')'$, where $\mathbf{Y}_i$ and $\boldsymbol{\Delta}_i$ has dimensions $n_i \times 1$ and $n_i \times m$, respectively, and $n_1 + n_2 = n$. Then, for $i = 1, 2$, $\mathbf{Y}_i \sim LCFUSN_{n_i,m}(\boldsymbol{\Delta}_i)$ with pdf given by

$$f_{\mathbf{Y}_i}(\mathbf{y}_i) = 2^m \left(\prod_{j=1}^{n_i} y_j\right)^{-1} \phi_{n_i}(\ln \mathbf{y}_i)\Phi_m(\boldsymbol{\Delta}_i' \ln \mathbf{y}_i|\mathbf{I}_m - \boldsymbol{\Delta}_i'\boldsymbol{\Delta}_i), \mathbf{y}_i \in \mathbf{R}^{n_i^+}. \tag{5}$$

However, we also proved (not shown) that it is not closed under conditioning.

3.Shannon entropy the CFUSN and log-CFUSN families

Despite their great contribution, Arellano-Valle and Genton (2005) did not obtain any result related to the entropy in the CFUSN family of distributions. To simplify notation, along this paper we assume that $\mathbf{\Delta}^* = \mathbf{I}_m - \mathbf{\Delta}'\mathbf{\Delta}$.
If $\mathbf{X} \sim CFUSN_{n,m}(\boldsymbol{\mu}, \mathbf{\Sigma}, \mathbf{\Delta})$, then the entropy of the canonical fundamental skew normal random vector $\mathbf{X}$ is

$$H_{CFUSN(\boldsymbol{\mu},\mathbf{\Sigma},\mathbf{\Delta})} = \frac{n}{2}\ln 2\pi + \frac{1}{2}\ln|\mathbf{\Sigma}| + \frac{1}{2}\sum_{i=1}^{n} E(X_{i0}^2) - E_{X_0}[\ln(2^m\Phi_m(\mathbf{\Delta}'\mathbf{X}_0|\mathbf{\Delta}^*))], \tag{6}$$

where X_{i0} is the ith component of the random vector $\mathbf{X}_0 \sim CFUSN_{n,m}(\mathbf{\Delta})$.
The entropy of the standard CFUSN family of distributions is concave and presents symmetric behavior around zero. The maximum entropy is obtained whenever the skewing parameter is zero, that is, in the normal case. We also noticed that the entropy decay is smooth for small values of m. Fixing the same $\mathbf{\Delta}$, we noticed that the entropy is higher if m is small. We also noticed that the entropy tends to be closer to the normal entropy for values of $\mathbf{\Delta}$ around zero.
The entropy of the log-CFUSN distribution is obtained similarly and is related with that given in (6). If $\mathbf{Z} \sim LCFUSN_{n,m}(\boldsymbol{\mu}, \mathbf{\Sigma}, \mathbf{\Delta})$, then the entropy of $\mathbf{Z}$ is

$$H_{LCFUSN_{n,m}(\boldsymbol{\mu},\mathbf{\Sigma},\mathbf{\Delta})} = H_{CFUSN_{n,m}(\boldsymbol{\mu},\mathbf{\Sigma},\mathbf{\Delta})} + \sum_{i=1}^{n} E(X_i), \tag{7}$$

where X_i is the ith component of the random vector $\mathbf{X} \sim CFUSN_{n,m}(\boldsymbol{\mu}, \mathbf{\Sigma}, \mathbf{\Delta})$.
For the $LCFUSN_{1,2}(\delta\mathbf{1_{1,2}})$ we noticed that the entropy increases with $\mathbf{\Delta}$ and the entropy is smoothy for small values of m. Moreover, for positive values of δ, the highest the m, the smallest the entropy. The opposite is observed for negative values of δ.

4. Bayesian inference in the Log-CFUSN family

In this section we discuss the estimation of the entropy in the log-CFUSN family of distributions under the Bayesian paradigm. Only some particular univariate cases are discussed since one of our goals in Section 5 is to evaluate the effect of increasing the dimension of the skewing function in data fitting. Similar strategy can be used in the multivariate case.
As can be notice from results in the previous section, the Shannon entropy depends on the parameters of the log-CFSUN distribution and thus, under the Bayesian paradigm, is random quantity. To estimate such quantity it is necessary to obtain their posterior distributions, which can be a hard task if we search for closed expressions for them. However, good approximations can be obtained using MCMC methods.
To achieve our goal, let $Y_1, ..., Y_L$ be a random sample of $Y \mid \mu, \sigma, \mathbf{\Delta} \overset{iid}{\sim} LCFUSN_{1,m}(\mu, \sigma^2, \mathbf{\Delta})$, which induces the following likelihood function:

$$\begin{aligned} f(\mathbf{y}|\mu,\sigma^2,\mathbf{\Delta}) &= 2^{Lm}(2\pi\sigma^2)^{-L/2}\left(\prod_{i=1}^{L} y_i\right)^{-1} \exp\left\{-\sum_{i=1}^{L}\frac{(\ln y_i - \mu)^2}{2\sigma^2}\right\} \\ &\times \prod_{i=1}^{L}\Phi_m(\mathbf{\Delta}'\sigma^{-1}(\ln y_i - \mu)|\mathbf{I}_m - \mathbf{\Delta}'\mathbf{\Delta}). \end{aligned} \tag{8}$$

If the population has a log-CFSUN distribution, it is not an easy task to sample from the posterior distributions without using a stochastic representation of the log-CFSUN family. Besides, it is not easy to elicit a prior distribution for the skewness parameter $\mathbf{\Delta}$ when we assume a very general structure for it. Let us consider, *a priori*, that μ, σ and $\mathbf{\Delta}$ are independent and such that $\mu \sim N(\mu_0, v)$ and $\sigma^2 \sim IG(\alpha, \beta)$, where $\mu_0 \in \mathbf{R}$, v, α and β are non-negative numbers. Consider a more parsimonious model by assuming $\mathbf{\Delta} = \delta\mathbf{1}_{1,m}$, where δ is a real number in the interval $(-1, 1)$, and elicit a uniform prior distribution for δ.

In order to sample from the posterior distributions let us consider the following data augmentation strategy. Notice that if $Y_i \sim LCFUSN_{1,m}(\mu, \sigma^2, \delta \mathbf{1}_{1,m})$ then $Z_i = \ln Y_i \sim CFUSN_{1,m}(\mu, \sigma^2, \delta \mathbf{1}_{1,m})$ and consider the stochastic representation of the CFUSN family Arellano-Valle and Genton (2005) given by

$$Z_i \stackrel{d}{=} \delta\sigma \mathbf{1}_{1,m}|\mathbf{X}_i| + [\sigma^2(1-\delta^2)]^{1/2} V_i + \mu, \tag{9}$$

where $\mathbf{X}_i \sim N_m(\mathbf{0}, \boldsymbol{I}_\mathbf{m})$, $V_i \sim N(0,1)$, $\mathbf{X}_i$ and V_i are independent random quantities and $|\mathbf{X}_i| = (|X_{i1}|, ..., |X_{im}|)'$. Consequently, we can hierarchically represent our model as

$$Y_i = \exp Z_i; \quad Z_i|\mathbf{X}_i = \mathbf{x}_i \sim N(\mu + \delta\sigma \mathbf{1}_{1,m}|\mathbf{X}_i|, \sigma^2(1-\delta^2)); \quad \mathbf{X}_i \sim N_m(\mathbf{0}, \boldsymbol{I}_\mathbf{m}), \tag{10}$$

where $\mathbf{X}_i$ is a latent (unobserved) random vector. Under such model representation it follows that the full conditional distributions for the parameter μ, σ^2 and δ and for the latent vector $\mathbf{X}_i$, $i = 1, \ldots, L$ are, respectively,

$$\begin{aligned}
\mu \mid \sigma^2, \delta, \mathbf{Z}, \mathbf{X} &\sim N\left(\frac{\mu_0\sigma^2(1-\delta^2) + v\sum_{i=1}^L (z_i - \sigma\delta\sum_{j=1}^m |x_{ij}|)}{Lv + \sigma^2(1-\delta^2)}, \frac{\sigma^2 v(1-\delta^2)}{Lv + \sigma^2(1-\delta^2)}\right), \\
f(\sigma^2 \mid \mu, \delta, \mathbf{Z}, \mathbf{X}) &\propto \left(\frac{1}{\sigma^2}\right)^{\alpha + L/2 + 1} \exp\left\{\frac{\sum_{i=1}^L 2(z_i - \mu)\delta\sum_{j=1}^m |x_{ij}|}{2\sigma(1-\delta^2)}\right\} \\
&\quad \times \exp\left\{-\frac{2\beta(1-\delta^2) + \sum_{i=1}^L 2(z_i - \mu)^2}{2\sigma^2(1-\delta^2)}\right\}, \\
f(\delta \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}, \mathbf{Z}, \mathbf{X}) &\propto \left(\frac{1}{1-\delta^2}\right)^{L/2} \exp\left\{-\frac{\sum_{i=1}^L (z_i - \mu - \delta\sigma\sum_{j=1}^m |x_{ij}|)^2}{2\sigma^2(1-\delta^2)}\right\}, \\
f(\mathbf{X}_i \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}, \mathbf{Z}, \delta \mathbf{X}_{(-i)}) &\propto \exp\left\{\sum_{i=1}^L \left[(z_i - \mu - \sigma\delta\sum_{j=1}^m |x_{ij}|)^2 - \frac{\sum_{j=1}^m x_{ij}^2}{2}\right]\right\}.
\end{aligned}$$

The Gibbs sampler can be used to sample from the posterior full conditional distribution of μ. The posterior full conditional distributions of σ, δ and $\mathbf{X}_i$, $i = 1, \ldots, n$, do not have closed forms. The Metropolis-Hastings algorithm can be used to sample from such distributions. Moreover, the hierarchical representation in (10) also allow us to use the software Winbugs to obtain samples from the posterior distributions.
For each sample $(\mu^t, (\sigma^2)^t, \delta^t)$ of the posterior distribution of (μ, σ^2, δ) we can obtain a sample of the posterior of the Shannon entropy $H_Y(\mu, \sigma^2, \delta)$ which is given in (7). Posterior summaries of $H_Y(\mu, \sigma^2, \delta)$ such as means, modes and HPDs can be approximated in the usual way.
Another way to estimate the Shannon entropy is to plug the posterior point estimates (usually, posterior means or modes) in expression of $H_Y(\mu, \sigma^2, \delta)$. One disadvantage of this procedure is that the posterior uncertainty about the parameters is not taken into consideration in the estimation of $H_Y(\mu, \sigma^2, \delta)$.

4. Selecting model using Shannon entropy

In this section we re-analyze the USA monthly precipitation data recorded from 1895 to 2007, that is available at the National Climatic Data Center (NCDC). It consists of 1.344 observations of the US precipitation index (PCL). Our goal here is to consider the Jaynes's (Jaynes, 1957) principle to select the best model, that is, we select the model that maximizes the Shannon entropy. Models are also chosen using two well-known tools for model selection, the conditional predictive ordinate (SlnCPO) and the deviance information criterion (DIC). Denote by Y_i the precipitation index in the ith month. We will assume different log-CFSUN family of distributions and evaluate the gain in assuming a higher dimensional skewing function to analyse the data, that is, we consider that $Y_i \mid \mu, \sigma^2, \boldsymbol{\Delta} \sim LCFUSN_{1,m}(\mu, \sigma^2, \delta \mathbf{1}_{m,1})$ and assume flat prior distributions for all parameters by eliciting $\mu \sim N(0, 100)$, $\sigma^2 \sim IG(0.1, 0.1)$ and $\delta \sim U(-1, 1)$. We also let m to vary from

$m = 1$ to $m = 5$. We name M_i the model for which we assume $m = i$. By considering such specifications and assuming the posterior means, we obtained very close plug-in estimates of the true density for all m as can be noticed in Figure 1.

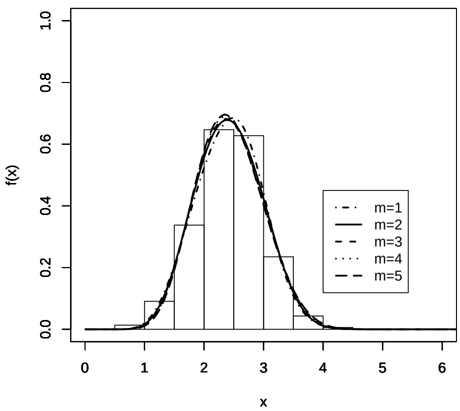

Figure 1: Fitted log-CFUSN densities, precipitation data.

Table 1 shows some summaries of the posterior distributions of all parameters. The posterior means for μ and σ^2 are similar for all models and increase as m increases. Also, all models point out a negative skewness in the data and the highest estimate for δ is obtained if $m = 1$, that is, whenever a less dimensional skewing function is assumed. It is also noteworthy that the posterior inference about μ is less precise for models with high m since the posterior variances for that parameter become higher as m increases. The opposite is observed for σ^2 and δ. The 95% HPD intervals disclose strong evidence in favour of an asymmetric model with negative skewness.

Table 1: Posterior summaries, Precipitation data

	μ		σ		δ		
m	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.	95%HPD
1	1.140	0.010	0.375	0.011	-0.947	0.010	$[-0.962, -0.925]$
2	1.276	0.013	0.384	0.010	-0.686	0.005	$[-0.694, -0.674]$
3	1.392	0.016	0.392	0.010	-0.570	0.004	$[-0.575, -0.561]$
4	1.483	0.015	0.394	0.008	-0.497	0.003	$[-0.499, -0.490]$
5	1.562	0.015	0.394	0.008	-0.446	0.001	$[-0.447, -0.441]$

Table 2 shows the 95% HPD, SlnCPO, DIC and the Shannon entropy comparing all models. The Shannon entropy was computed using the two different ways discussed in Section 4.

We notice that the mean Shannon entropy increases with the complexity of the model. Moreover, the HPDs for models M_1 and M_2 point out that the amount of information in our system is quite similar and this same conclusion can be drawn for models M_2 and M_3. It is important to observe that the HDP's for models M_4 and M_5 do not intersect, making these two models significantly different. By using the maximum entropy principle we decide for M_5 as the best model, and this decision is the same we make using usual procedures for model selection, such as DIC and the SlnCPO. Moreover, we notice that, at least in this example, the Shannon entropy and the DIC have similar behavior and leads to the same decision.

Table 2: Model selection statistics, Precipitation data

m	$Mean Entropy$	95% HPD	Plug-in Entropy	DIC	SlnCPO
1	1.143	$[0.963; 1.313]$	1.158	$-\ 13,190$	-0.83766
2	1.164	$[1.101; 1.236]$	1.230	$-\ 36,960$	-0.83545
3	1.256	$[1.160; 1.348]$	1.314	$-112,400$	-0.83765
4	1.467	$[1.364; 1.566]$	1.539	$-321,100$	-0.84144
5	1.777	$[1.675; 1.880]$	1.815	$-895,300$	-0.81057

5. Conclusions

We introduced the log-CFUSN family of distributions and studied some of its properties such as marginal distribution and Shannon entropy. We also discussed some issues related to Bayesian inference in that family. Analizing the USA precipitation dataset, we concluded that the use of a skewing function with higher dimension than that assumed by Marchenko and Genton (2010) can bring some gain to the model fit.
The main motivation for studying the log-CFUSN family of distribution in detail is the result that appeared in Santos et al. (2013) where it was shown that such family is of fundamental interest in the interpretation of the parameters in mixed logistic regression model if the random effects are skew-normally distributed. In that paper it was proved that, under skew-normality, the odds ratio has distribution in the log-CFUSN family. More properties of the log-CFUSN family of distribution, such as moments, conditional distributions, stocastic representations relative entropy and its relation with other families of distributions can be found in Queiroz (2013).

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