

# **A Non Parametric Approach to Estimating Joint Maternal and Child Survival**

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## **Abstract**

Since the adoption of the Millennium Development Goal in 2000, much interest has been given to the study of maternal and child health. Despite several linkages among them, maternal and child survival continue to be studied independently. In this study, we apply survival analysis techniques and basic probability laws, to develop protective tables for the joint maternal and child hazards and survivals. Applying this technique to data from Ghana, the joint probabilities show that, about 93 percent of labouring women are expected to independently survive, up to one month after delivery, with their child; about 3 percent of mothers are expected to lose their children by the end of one month after delivery, and about 5 percent of children will lose their mothers, by one month after delivery. The results indicate that maternal mortality is higher than child mortality within the first one month after delivery.

**Keywords:** Joint probability; Maternal Survival; Child Survival; hazard.

## **1. Introduction**

Improving maternal and child health have been key concerns for many nations and these are evident from observations from numerous international summits and conferences including the Millennium Summit in 2000. Two of the eight Millennium Development Goals (MDGs) adopted at the Millennium Summit are on improving child health (MDG4) and maternal health (MDG5).

The health and well-being of women and their children are completely linked (Bhutta, et al., 2008). Indeed our ability to reduce child mortality can be said to depend largely on mothers. For example maternal attributes, which are biological factors of birth, like the age of the mother at the time of childbirth, birth order and birth interval (Forste, 1994; Rutstein, 1984), are shown to all have significant effects on child survival. Infants born to mothers who have lost a child are at greater risk of dying during infancy (Cleland & Van Ginneken, 1988). Some studies also show that, within a family, deaths of infants are correlated (Curtis, Diamond & McDonald, 1993; Das Gupta, 1990; Gubhaju, 1985; Majumder, 1989). The relationship between child survival and mothers are several (Cleland & Van Ginneken, 1988; Tinker & Ransom, 2002). Thus any attempt to reduce child mortality without a corresponding attention to mothers could be a fiasco.

Despite the much suggested linkage between maternal and child mortality, much of the studies in this area have focused on them separately. At best, maternal factors have been used as covariates in the study of child mortality or vice versa. These types of study only provide suggested linkages but do not adequately describe the nature of the relationship between these two issues.

The need to study maternal and child survival jointly is imperative: The suggested linkage between these, require study into their relationship together. The study on joint hazard and survival for mother and child is therefore an attempt to develop a non parametric estimate which describes the survival of both mother and child together after labor sets in to one month after delivery.

## **2. Methodology**

Basic theory in probability is used to complement non parametric survival analysis techniques in developing the joint probabilities of mother and child survival to a particular time. This is possible because the survival functions,  $S(t)$ , is a probability function and therefore yields itself to the laws of probability; therefore it can be used to find joint probabilities. Similarly, the hazard function,  $h(t)$ , is also a probability function.

## 2.1 Non Parametric Survival Analysis

Let  $T$ , denote the survival time from onset of Labour. The distribution of  $T$  can be characterized by three equivalent functions, Survival Function,  $S(t)$ , Probability Density function,  $f(t)$ , and the Hazard function,  $h(t)$  (Lee & Wang, 2003).

$S(t) = P(\text{a woman/child surviving longer than time } t \text{ from onset of labour})$

$$f(t) = \frac{\lim_{\Delta t \rightarrow 0} P[\text{a woman/child dying in the interval } (t, t+\Delta t) \text{ after onset of labour}]}{\Delta t}$$

$$h(t) = \frac{\lim_{\Delta t \rightarrow 0} P[\text{a woman/child dying in the interval } (t, t+\delta t) \text{ given that the woman/child survived to } t]}{\Delta t}$$

where  $h(t) = \frac{f(t)}{S(t)}$

these functions were estimated using the Life table method (Gehan, 1969):

For the  $i^{th}$  interval, let  $t_i$  be the end time and  $q_i$  be the conditional probability of dying. Then;

$$\hat{S}(t_i) = \prod_{j=1}^{i-1} (1 - \hat{q}_j)$$

$$\hat{f}(t_{mi}) = \frac{\hat{S}(t_i) - \hat{S}(t_{i-1})}{b_i} = \frac{\hat{S}(t_i) \hat{q}_i}{b_i}$$

$$\hat{h}(t_{mi}) = \frac{d_i}{b_i(n_i - \frac{1}{2}d_i)} = \frac{2\hat{q}_i}{b_i(1 - \hat{p}_i)}$$

Where

$t_{mi}$  is the mid-point of the  $i^{th}$  interval,

$d_i$  is the number of women dying in the  $i^{th}$  interval after their conception,

$n_i$  is the number of women exposed in the  $i^{th}$  interval after conception,

$q_i = \frac{d_i}{n_i}$  is the conditional probability of a woman dying in the  $i^{th}$  interval after conception

$p_i = (1 - q_i)$  is the conditional probability of a woman dying in the  $i^{th}$  interval after conception

$b_i$  is the width of the  $i^{th}$  interval

The standard error of the survival function, Greenwood (1926) is estimated by:

$$s.e.(\hat{S}(t_i)) \approx \hat{S}(t_i) \sqrt{\sum_{j=1}^{i-1} \frac{\hat{q}_j}{n_j(1-\hat{q}_j)}}$$

While that of the hazard function, Gehan (1969), is estimated by;

$$s.e.(\hat{h}(t_{mi})) \approx \hat{h}(t_{mi}) \sqrt{\frac{\{1 - [\frac{1}{2}\hat{h}(t_{mi})b_i]^2\}}{n_i\hat{q}_i}}$$

The probability density function, Gehan (1969), is estimated by;

$$s. e. (\hat{f}(t_{mi})) \approx \hat{S}(t_i) \hat{q}_i \sqrt{\frac{\left( \sum_{j=1}^{i-1} \frac{\hat{q}_j}{n_j(1-\hat{q}_i)} + \frac{(1-\hat{q}_i)}{n_j \hat{q}_j} \right)}{b_i}}$$

## 2.2 Basic Probability Theory used in developing the Joint probabilities

Let  $A$  and  $B$  be defined events in a sample space  $S$ : then from basic probability theory, if  $A$  and  $A'$  are complementary events in the sample space  $S$ , then

$$P(A') = 1 - P(A) \quad (1)$$

Also, if  $A$  and  $B$  are independent, then

$$P(A \cap B) = P(A) \times P(B) \quad (2)$$

In this study we let  $C$  represent child and  $M$  represent maternal. If we define  $P(C)$  to be the probability that a Child survives from onset of labour to time  $t$  and  $P(M)$  to be the probability that a Mother survives from onset of labour to the time  $t$ ; and also define  $P(M \cap C)$  to be the probability that a woman and child survive from onset of labour to the time  $t$ . Then from equation 2, assuming independence of  $M$  and  $C$ , at any point in time  $t$

$$P(M \cap C) = P(M) \times P(C) \quad (3)$$

$$P(M \cap C') = P(M) \times P(C') \quad (4)$$

$$P(M' \cap C) = P(M') \times P(C) \quad (5)$$

$$P(M' \cap C') = P(M') \times P(C') \quad (6)$$

The joint hazard is obtained using the same rules following similar definitions made for the hazard.

## 3. Results

Survival and hazard estimates are shown in tables 1 and 2 with the joint probabilities shown in Table 3 and 4. Table 3 shows that the chances of a mother and child independently surviving to 30 days after delivery is about 93 percent (92.5414), mother surviving independently without her child is about 3 percent (2.756) and a child independently surviving without the mother is about 5 percent (4.566). The results indicate that the chances of a laboring mother independently dying within one month after delivery is higher than that for a child: Table 4 shows that the highest risk is mortality is highest at the beginning (day 1) and reduces with time.

**Table 1. Survival Estimates for Neonate and Mother after Labour**

<i>Time (Days)</i>	<b>Survival</b>			
	<i>Neonatal</i>	<i>S.E.</i>	<i>Maternal</i>	<i>S.E.</i>
2	0.98457	0.00141	0.96957	0.00202
4	0.97933	0.00163	0.96404	0.00219
6	0.97645	0.00173	0.96183	0.00225
8	0.97383	0.00183	0.96017	0.00230
10	0.97357	0.00183	0.95989	0.00231
12	0.97305	0.00185	0.95947	0.00232
14	0.97292	0.00186	0.95920	0.00233
16	0.97187	0.00189	0.95602	0.00241
18	0.97187	0.00189	0.95588	0.00242
20	0.97187	0.00189	0.95560	0.00242
22	0.97134	0.00191	0.95367	0.00247
24	0.97134	0.00191	0.95367	0.00247
26	0.97134	0.00191	0.95367	0.00247
28	0.97121	0.00191	0.95353	0.00248
30	0.97108	0.00192	0.95297	0.00249

**Table 2. Joint Survival Estimates for Neonate and Mother after Labour**

<i>Time (Days)</i>	<b>Joint Survival</b>			
	<i>M &amp; C</i>	<i>M &amp; C'</i>	<i>M' &amp; C</i>	<i>M' &amp; C'</i>
2	0.954611	0.014960	0.029959	0.000470
4	0.944112	0.019927	0.035218	0.000743
6	0.939175	0.022651	0.037275	0.000899
8	0.935038	0.025128	0.038792	0.001042
10	0.934519	0.025370	0.039051	0.001060
12	0.933616	0.025858	0.039434	0.001092
14	0.933223	0.025975	0.039697	0.001105
16	0.929124	0.026893	0.042746	0.001237
18	0.928989	0.026889	0.042881	0.001241
20	0.928721	0.026881	0.043149	0.001249
22	0.926333	0.027332	0.045007	0.001328
24	0.926333	0.027332	0.045007	0.001328
26	0.926333	0.027332	0.045007	0.001328
28	0.926075	0.027452	0.045135	0.001338
30	0.925414	0.027560	0.045666	0.001360

**Table 3. Hazard Estimates for Neonate and Mother after Labour**

<i>Time (Days)</i>	<b>Hazard</b>			
	<i>Neonatal</i>	<i>S.E.</i>	<i>Maternal</i>	<i>S.E.</i>
1	0.00778	0.00072	0.03043	0.00202
3	0.00266	0.00042	0.00571	0.00090
5	0.00147	0.00031	0.00230	0.00057
7	0.00134	0.00030	0.00173	0.00050
9	0.00013	0.00010	0.00029	0.00020
11	0.00027	0.00013	0.00043	0.00025
13	0.00007	0.00007	0.00029	0.00020
15	0.00054	0.00019	0.00332	0.00069
17	0.00000	*	0.00014	0.00014
19	0.00000	*	0.00029	0.00020
21	0.00027	0.00014	0.00203	0.00054
23	0.00000	*	0.00000	0.00000
25	0.00000	*	0.00000	0.00000
27	0.00007	0.00007	0.00015	0.00015
29	0.00007	0.00007	0.00058	0.00029

**Table 4. Joint Hazard Estimates for Neonate and Mother after Labour**

<i>Time (Days)</i>	<b>Joint Hazard</b>			
	<i>M &amp; C</i>	<i>M &amp; C'</i>	<i>M' &amp; C</i>	<i>M' &amp; C'</i>
1	0.00023674	0.0301921	0.0075433	0.9620279
3	0.00001518	0.0056909	0.0026448	0.9916491
5	0.00000338	0.0022922	0.0014666	0.9962378
7	0.00000231	0.0017233	0.0013377	0.9969367
9	0.00000004	0.0002881	0.0001300	0.9995819
11	0.00000012	0.0004322	0.0002699	0.9992978
13	0.00000002	0.0002883	0.0000700	0.9996417
15	0.00000179	0.0033147	0.0005382	0.9961453
17	0.00000000	0.0001447	0.0000000	0.9998553
19	0.00000000	0.0002894	0.0000000	0.9997106
21	0.00000055	0.0020258	0.0002695	0.9977042
23	0.00000000	0.0000000	0.0000000	1.0000000
25	0.00000000	0.0000000	0.0000000	1.0000000
27	0.00000001	0.0001450	0.0000700	0.9997850
29	0.000000041	0.0005802	0.0000700	0.9993498



#### 4. Conclusion

The need to heighten attention on a laboring woman, without neglecting her child, has been highlighted in this study. Perhaps the notion that a woman is in a better position to take care of herself, than her baby can, might be influencing the lesser attention given to the mothers and thereby increasing their risk.

#### 5. Future Works

This study focused on the assumption of independence of maternal and child survival. Ongoing work is targeted at assessing the validity of this assumption and how the possible impact of maternal mortality on child survival. Future work will however target developing the joint distribution and the stochastic approach this joint phenomenon since every mother (and child) can be categorised into one of four categories at any stage, and can also progress from one category to another with time (which could be a different state).

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