Improving bias in nonparametric density estimation: $L_1$ view

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Density estimation is important for many applications, including densities of returns on financial assets. Researchers are concerned with the precision of estimation (bias) while keeping variability of estimators low. There are well-established results regarding the order of bias, when the density is sufficiently smooth and the kernel used for estimation is of a corresponding order. In this paper we show that two of these results can be improved further. We define a new kernel density estimator and show that it improves the existing convergence estimates when the error is measured in the space of integrable functions. In one result, the bias is shown to be $o(h^s)$ instead of the conventional $O(h^s)$, assuming that the density is $s$-smooth and the kernel is of order $s$. In the other result, for a density whose $s$th order derivative is $\alpha$-Lipschitz, we prove that the bias is of order $h^{s+\alpha}$, provided that the kernel order is $s$, while the extant papers require "larger than $s".

Keywords: Rosenblatt-Parzen estimator; convergence; higher order kernel.