MGARCH models: Tradeoff between feasibility and dynamic dependencies in volatilities and covariances

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Abstract

The multivariate GARCH (MGARCH) models are popular to represent conditional second-order moments of conditionally heteroscedastic multivariate time series. The original MGARCH models were strongly restricted to reduce the number of parameters to make their estimation feasible in systems with a large number of series and to guarantee positive definiteness of the conditional covariance matrices. However, these restrictions limit the dynamics that MGARCH models can represent, assuming for example, that the volatilities evolve in a univariate fashion in such a way that they are not related either among them nor with the correlations. This paper focuses on the limitations implied by the restrictions usually imposed in practice in symmetric MGARCH models. The models are illustrated using simulated and empirical data.

Keywords: multivariate GARCH models; multivariate volatility models; restrictions in MGARCH models

1. Introduction

The univariate generalized autoregressive conditional heteroscedastic (GARCH) models were soon extended to a multivariate framework by Bollerslev *et al.* (1988). Since then, multivariate GARCH (MGARCH) models have attracted a great deal of attention due to the many applications that require estimates of conditional variances and covariances of multivariate time series. Generally, MGARCH models appear in the context of systems of financial returns; see, for example, Bollerslev *et al.* (1988), Andersen *et al.* (2007), Engle and Kelly (2012) and Santos *et al.* (2013), for a few selected asset pricing, portfolio selection, risk management and future hedging applications. Depending on the particular application of MGARCH models, the number of returns in the system can vary from rather small (Bollerslev *et al.* , 1988; Beirne *et al.*, 2013) to extremely large (Santos *et al.*, 2013; Rombouts *et al.*, 2014).

The original MGARCH models were fairly flexible in the sense of allowing all volatilities and conditional correlations in the model to be related with each other. However, in practice, the empirical implementation of MGARCH models was limited due to two main limitations. First, due to the need to estimate a large number of parameters, the implementation of MGARCH models was originally restricted to systems with very few series, usually two. For large systems, their parameters had to be heavily restricted to reduce the dimensionality and make estimation a feasible task. Second, the parameters of MGARCH models need to be restricted to guarantee the positive definiteness of conditional covariance matrices, which is not always straightforward. Consequently, the most popular MGARCH models often implemented to represent the dynamic evolution of volatilities and correlations of real time series systems restrict their dynamics in a such a way that the estimation of the model parameters is feasible and the conditional covariance matrices are guaranteed to be positive definite. Most of these restrictions are based on assumptions that volatilities depend on their own past values without interrelations neither among volatilities nor between volatilities and correlations. However, if these restrictions are not approximately satisfied in reality, the estimated volatilities

Acknowledgments: [1]CAPES, Brazil, grant 0969/13-3; [2]CAPES, grant 10600/13-2 and FAPESP, grant 2013/00506-1; [3]Grant ECO2012-32401 from a government project.

and correlations may suffer from strong biases; see Ledoit *et al.* (2003), Rossi and Spazzini (2010) and Amado and Teräsvirta (2014) for some consequences of misspecifying conditional variances and covariances. Audrino (2006) and Laurent *et al.* (2012) focus on the question of how to choose a GARCH-type model for the individual conditional variances in different MGARCH settings and its relevance and impact on the final accuracy of the conditional covariance estimates. Engle (2009) argues that although it seems important to allow for square and cross-products of one asset to help forecasting variances and covariances of other assets, in fact, there are few clear examples of these interrelations in the literature. On the contrary, several works imply that allowing for interrelations among conditional variances and correlations may be important. First, the empirical evidence on volatility feedbacks is plentiful. Recently, Beirne *et al.* (2013) found evidence of these feedbacks; see Nakatani and Teräsvirta (2009) and Tsiaplias and Chua (2013) for further empirical evidence of volatility feedbacks. Therefore, it seems empirically important to allow for volatility interactions when specifying MGARCH models. Second, it has often been found that volatilities and cross-correlations across assets move together over time so they cannot be estimated separately. For example, Ramchand and Susmel (1998), Longin and Solnik (2001) and Okimoto (2008) find that cross-correlations between markets are higher during unstable periods. See Bauwens *et al.* (2006) and Bauwens and Otranto (2013) for a more complete survey.

The choice of a multivariate model can lead to different conclusions in any application that involves forecasting dynamic covariance matrices. However, most multivariate time-varying covariance models are often chosen on an ad hoc basis. In this paper, we analyze the empirical implications of the restrictions imposed on MGARCH models to reduce the number of parameters and/or to guarantee positiveness. A Monte Carlo experiment is presented in Section 2, an empirical application is presented in Section 3 and Section 4 concludes.

2. Simulation Study

Here we carry out a Monte Carlo experiment is carried out to analyze finite sample properties of the estimated conditional variances, covariances and correlations after fitting a restricted specification to systems in which all variances and covariances are interrelated with each other. We also consider alternative estimators to estimate the pseudo-parameters in the restricted specifications considered. We simulate 500 replications of sizes $T = 1000$ and 2000 considering Gaussian and 7-degree-freedom Student-t innovations. For each replicate, we estimate the VECH, DVECH (diagonal VECH), BEKK(1), DBEKK(1) (diagonal BEKK), SBEKK(1) (scalar BEKK), D-RBEKK (rotated BEKK), S-RBEKK, CCC, ECCC, cDCC and RDCC models, all of them including just one lag in relation to the past returns, conditional variances and covariances. The VECH model is estimated by (Q)ML-G ((quasi) maximum Gaussian likelihood), (Q)ML-S ((quasi) maximum Student-t likelihood) and variance targeting (VT) methods. The DVECH, BEKK(1), DBEKK(1) and SBEKK(1) models are estimated by the QML-G, QML-S and by VT procedures. The DVECH model is also estimated by the two-step procedure of Ledoit *et al.* (2003). In the CCC, D-RBEKK and S-RBEKK models, we consider the Gaussian two-stage (2s-G) and Student two-stage (2s-S) estimation methods and in the cDCC and RDCC, the estimation is done by the Gaussian three-stage (3s-G) and Student three-stage (3s-S) methods. The parameters are not restricted to ensure positivity of the matrices of covariances and covariance stationarity of the model in the VECH and DVECH models, nor to ensure covariance stationarity of the BEKK(1) and DBEKK(1) models, because it is difficult to impose these restrictions that involves eigenvalues in the maximization. We restrict $a + b < 1$ in the cDCC and RDCC models and $a^2 + b^2 < 1$ in the S-RBEKK and SBEKK(1) models, which are sufficient conditions to ensure positivity of the covariance matrices. In the DBEKK(1) and D-RBEKK estimation, we impose $a_{ii}^2 + b_{ii}^2 < 1$, $\forall i$, in order to ensure positivity of the matrices of covariances matrices. For each replicate and estimator considered, the performance of the estimated conditional covariance matrices is measured, by using the following goodness of fit statistic

$$L_E = \frac{\sum_{t=51}^{T} \text{vech}(\hat{\mathbf{H}}_t - \mathbf{H}_t)'\text{vech}(\hat{\mathbf{H}}_t - \mathbf{H}_t)}{T - 50}, \quad (1)$$

where $\hat{\mathbf{H}}_t$ is the estimated covariance matrix at time t; see Laurent *et al.* (2013) for a comprehensive list of different loss functions and their impacts on ranking forecasting performances of MGARCH models. To establish which elements of the matrices are more accurately estimated, we compute the bias and mean square errors (MSE) of the conditional standard deviations, covariances and correlations as in Ledoit *et al.* (2003) and Audrino (2014). At the end of the simulation we evaluate, for all the models and estimation

Table 1: Summary of the Monte Carlo simulation: median of the biases and of MSE of estimated volatility, covariances and correlations and of the L_E statistics after fitting MGARCH models by different estimation procedures. The DGP is the Gaussian VECH process and $T = 1000$.

Model	Method	Median of bias in simulations				Median of MSE in simulations				L_E
		$\sqrt{h_1}$	$\sqrt{h_2}$	h_{12}	ρ_{12}	$\sqrt{h_1}$	$\sqrt{h_2}$	h_{12}	ρ_{12}	$\mathbf{H}_t$
Scale		$\times10^{-5}$	$\times10^{-5}$	$\times10^{-6}$	$\times10^{-2}$	$\times10^{-7}$	$\times10^{-7}$	$\times10^{-10}$	$\times10^{-2}$	$\times10^{-9}$
VECH	ML-G	-2.91	2.35	-0.41	1.89	4.42	3.91	2.36	0.30	1.17
	QML-S	-2.65	2.24	-0.28	0.004	3.79	3.19	1.66	0.28	1.02
	VT	-2.03	-0.006	-2.41	2.08	4.37	4.01	2.20	0.35	1.02
DVECH	QML-G	-0.002	0.004	-9.54	-2.21	8.35	7.33	5.85	0.42	2.15
	QML-S	-1.20	-0.87	-9.51	-2.04	7.97	7.63	5.73	0.42	2.14
	VT	-0.007	-1.89	-2.26	2.66	11.61	10.10	5.19	0.50	2.62
	Led	-0.53	-0.14	-11.13	-5.46	7.07	6.89	5.04	0.69	1.95
BEKK	QML-G	0.012	3.25	-1.10	-2.74	14.59	11.70	7.86	1.59	3.44
	QML-S	-1.86	3.31	-1.73	-2.34	14.07	10.42	7.72	1.52	3.22
	VT	-1.84	-0.003	-2.35	2.33	15.68	9.92	6.93	1.56	3.34
DBEKK	QML-G	-0.001	1.81	-0.46	-2.04	9.99	9.09	3.13	1.11	2.26
	QML-S	0.71	1.51	0.12	-1.32	9.91	9.25	3.27	1.11	2.22
	VT	1.62	0.003	-2.35	-2.70	10.24	9.03	2.84	1.14	2.25
SBEKK	QML-G	0.001	1.92	0.30	-2.08	7.77	7.62	3.44	1.25	1.72
	QML-S	1.12	1.81	1.03	-1.34	8.15	7.86	3.50	1.20	1.81
	VT	-1.76	-0.005	-2.36	-3.18	8.12	7.53	3.09	1.32	1.78
D-RBEKK	2s-G	-1.60	-1.73	-2.38	-3.02	9.17	8.34	2.89	1.25	2.04
	2s-S	-0.30	-0.14	-2.37	-2.97	9.51	8.71	2.85	1.21	2.12
S-RBEKK	2s-G	-1.72	-1.96	-2.36	-3.15	8.09	7.52	3.07	1.33	1.78
	2s-S	-0.36	-0.09	-2.36	-3.09	8.52	7.82	3.04	1.30	1.88
CCC	2s-G	-0.53	-0.14	0.62	1.79	7.07	6.89	3.48	0.85	1.77
	2s-S	-0.43	-0.16	1.26	-0.33	7.04	6.87	3.44	0.86	1.78
ECCC	2s-G	0.001	-0.003	3.28	0.06	5.19	4.86	4.25	0.87	1.54
	2s-S	-2.69	2.27	4.31	0.25	4.82	4.83	4.58	0.86	1.53
cDCC	3s-G	-0.53	-0.14	0.69	-0.74	7.07	6.89	1.39	0.48	1.55
	3s-S	-0.43	-0.16	0.67	-0.75	7.04	6.87	1.52	0.50	1.59
RDCC	3s-G	-0.53	-0.14	0.73	-0.78	7.07	6.89	1.41	0.48	1.54
	3s-S	-0.43	-0.16	0.87	-0.77	7.04	6.87	1.38	0.47	1.56

methods, the median of these statistics. Table 1 reports the median of the bias, MSE and LE statistics for the case of Gaussian innovations and sample size $T = 1000$. The other cases are not reported, but the results are similar. Because in all cases the bias is negligible, all the next comments are related to the performance measures of other goodness of fit statistics defined previously.

According to the MSE and L_E criteria, the VECH model provides the best fit for all the three estimation methods, (Q)ML-G, (Q)ML-S and VT, since they occupy the first three rank positions, except for a few cases. This result is expected since the series is generated by the VECH process. The few cases occur according to the MSE criterion: in two cases the VECH fits occupy three of the first four positions, and the other exception occurs in the estimation of the conditional covariances when $T = 1000$, where the cDCC and RDCC are the best ones. Although the VECH model is not always the best to estimate the conditional covariance, it always performs better in estimating of the conditional correlation. Among the misspecified models, the best ones to estimate the conditional variance, covariance and correlation, according to the MSE criterion, are the ECCC, cDCC and RDCC, and DVECH (QML-G, QML-S and VT), respectively. The models that fit the univariate volatilities by the GARCH(1,1) model, i.e., the DVECH by Ledoit, cDCC and RDCC fits are better, according to the MSE, than the DVECH (QML-G, QML-S and VT), BEKK(1) and its restricted versions and RBEKK models. This means that incorporating incorrect dependence specifications increases the errors in the volatilities. In general, the BEKK(1) model has the worst performance, followed by the DBEKK model, except for the estimation of the conditional covariances, where the DVECH model has the second worst performance. The performance of the BEKK(1) model is even worse than that of simple models such as the SBEKK(1) and the CCC. The rotated fits (S-RBEKK and D-RBEKK) have similar performance as the non-rotated versions (SBEKK(1) and DBEKK(1)). Because the estimation in the rotated cases is faster and ensures positivity of covariance matrices and stationarity, the rotated BEKK models are recommended rather than the non-rotated BEKK.

The performance of the QML-G, QML-S and VT estimators is very similar if the data are fitted by the same model, independently of whether the generated data are Gaussian or Student or if $T = 1000$ or 2000. The VT procedure has the advantage of being faster, but it does not ensure that the covariances matrices are positive definite. When the DGP is the Student VECH process, the estimates using Student VECH model is better than using Gaussian VECH model, and a similar result is obtained using 3s-S and 3s-G estimators in the cDCC and RDCC models. When the DGP is the Gaussian process, in general the estimates using the Student VECH model is comparable to the estimates using the Gaussian VECH model, and even better in some cases. The DVECH estimation by the Ledoit procedure has a greater error in relation to conditional covariances and correlations than the DVECH estimation by QML-G, QML-S and VT procedures.

Concluding, the restrictions imposed by a misspecified model are much more relevant than the choice of the estimation method. Moreover, all alternative models have notably inferior performance in relation to the VECH model, especially when $T = 2000$, and among the alternative models, the cDCC has, in general, superior performance. These results are in agreement with Laurent *et al.* (2012)'s empirical study.

3. Application to Empirical Data

We apply some MGARCH models to a system of daily log- returns, in percentages, of five currencies: Euro (EUR), British Pound (GBP), Swiss Franc (CHF), Australian Dollar (AUD) and Japanese Yen (JPY) against the US Dollar (USD), observed from January 2, 2004 to December 31, 2013 (2582 observations). The data are closing exchange rate at 12:00 am, New York, discarding weekends and 26 days where at least four currencies have missing values. Because the sample autocorrelations are very small, and generally not significant in the first two lags and multiple of five lags, we decided to fit only a constant plus MGARCH models. We compare the following models and estimators: the DVECH model using the Ledoit *et al.* (2003) two-step procedure, the SBEKK model by QML-G, QML-S and VT, the DBEKK model by VT and the D-RBEKK, S-RBEKK, CCC, cDCC and RDCC models considering Gaussian and Student likelihoods. The out-of-sample predictions are evaluated by the rolling window procedure, as in Giacomini and White (2006). The first in-sample estimation period is from January 2, 2004 to December 31, 2012, leaving 258 days for an out-of-sample period forecast. We use the out-of-sample negative log-likelihood (OS-NL) function to assess the adequacy of the forecasts. The OS-NL, which is used by Audrino (2014), does not consider any proxy for the unobservable covariance matrices. We use a slightly different definition. For the h-step ahead prediction, it is defined as

$$\text{OS-NL} = -\sum_{i=1}^{n_{pred,h}} \log[\mathbf{f}(\mathbf{r}_{t_i+h}; \hat{\theta}_i, \hat{\mathbf{H}}_{t_i}(h))], \tag{2}$$

where $\mathbf{f}(.;.)$ is the conditional density of the returns, given by the innovation distribution, $n_{pred,h}$ is the number of h-steps ahead predictions, $\mathbf{r}_{t_i}$ is the last observation in window i, $\hat{\theta}_i$ is the parameter estimate for window i, and $\hat{\mathbf{H}}_{t_i}(h)$ is the h-steps ahead prediction of $\mathbf{H}_{t_i+h}$ in window i. The prediction $\hat{\mathbf{H}}_{t_i}(h)$ is evaluated by the conditional expectation of $\mathbf{H}_{t_i+h}$ on observations up to the t_i evaluated at $\hat{\theta}_i$. We also present results of the application of the superior predictive ability (SPA) test of Hansen (2005) and the model confidence set (MCS) of Hansen *et al.* (2011) to verify whether the differences in the OS-NL values are significantly different. We compute the SPA test and MSC using the Sheppard MFE Toolbox package with 10,000 bootstrap replications and block length of 6. The results do not change with block length 3 or 9. The first test allows for multiple comparison against a pre-specified benchmark model, i.e., testing whether each model is statistically outperformed by at least one of the other competing models. The second one chooses from an initial set, a subset of forecasts that outperforms all the other alternatives according to some criterion at a confidence level α. The observed OS-NL values as well the p-values of the SPA tests and the MCS with $\alpha = 5\%$, 10% and 25%, for predicting one-, five- and twenty-step ahead covariance matrices considering different models and estimation methods are in Table 2. The SBEKK by QML-S and the S-RBEKK by 2s-S have the best performance for $h = 1$ and the RDCC by 3s-S for $h = 5$ and 20. The CCC by QML-G and DVECH by Ledoit are the worst fits. For all forecasting horizons, models with Student innovations

Table 2: Results based on the OS-NL for $h = 1$, 5 and 20 forecasting horizon for the empirical application. In the first panel, the OS-NL and the p-value of the SPA tests for the null hypothesis that each model is not outperformed by any other model. The models with p-value larger than 10% are in bold. The second panel contains the models selected by MCS at $\alpha = 5\%$, 10% and 25% significance levels.

Model	Method	1-step ahead		5-steps ahead		20-steps ahead	
		OS-NL	SPA	OS-NL	SPA	OS-NL	SPA
DVECH	Led	822.0	0.0000	822.7	0.0000	800.8	0.0000
SBEKK	QML-G	804.9	0.0065	823.1	0.0401	789.2	0.0005
	QML-S	**771.7**	1.0000	**773.9**	0.7910	744.3	0.4082
	VT	815.2	0.0010	820.8	0.0014	792.9	0.0000
DBEKK	VT	814.5	0.0147	828.2	0.0366	790.6	0.0003
S-RBEKK	2s-G	815.2	0.0013	820.8	0.0025	792.9	0.0001
	2s-S	**777.9**	0.2241	**774.1**	0.7434	747.6	0.1451
D-RBEKK	2s-G	818.7	0.0111	845.5	0.0649	821.6	0.0034
	2s-S	**779.7**	0.1061	**784.5**	0.1421	779.5	0.0597
CCC	2s-G	876.7	0.0012	861.0	0.0009	818.5	0.0002
	2s-S	811.9	0.0471	**799.2**	0.0675	755.1	0.1390
cDCC	3s-G	823.4	0.0055	826.5	0.0079	788.2	0.0001
	3s-S	**781.0**	0.1162	**775.3**	0.2053	742.3	0.0127
RDCC	3s-G	821.3	0.0061	823.8	0.0172	782.8	0.0000
	3s-S	**780.0**	0.1439	**772.9**	1.0000	**738.2**	1.0000

h	α	MCS - models selected
1	any level	SBEKK, R-SBEKK, R-DBEKK, cDCC and RDCC with Student
	only $\alpha \leq 10\%$	All models with Student innovations, except RDCC with Student
5	any level	All models with Student innovations
	only $\alpha \leq 10\%$	SBEKK-QML-G, D-RBEKK 2s-G
	only $\alpha \leq 5\%$	DBEKK-VT
20	any level	RDCC with Student

have better performance than models with Gaussian innovations; few models with Student innovations are significantly outperformed in the SPA test at 5% confidence level. The exceptions occur in the CCC 2s-S for $h = 1$ and the cDCC 2s-S for $h = 20$, but they are not outperformed at the 1% level. Furthermore, the MCS at 5% and 10% levels selects all models with Student innovations for $h = 1$; at 25% level the CCC model with t-Student innovations is excluded and the others continue in the MCS. For $h = 5$, the MSC includes all the models with Student innovations, the SBEKK QML-G, DBEKK VT and D-RBEKK 2s-G at 5% level and all the models with Student innovations at the 25% level. For $h = 20$, the RDCC with Student innovation is the only model selected in the MCS at any one of the three levels considered.

4. Conclusions

In this paper we discuss the main strengths and limitations of the most popular symmetric multivariate GARCH models available in the literature. A simulation study is carried out to compare different specifications and estimators, when the series are generated by the general VECH process. The DVECH and DCC models explain the general model very well, whereas the BEKK(1) fails to estimate conditional covariances and correlations. An empirical application to a system of five exchange rate returns is carried out. Much progress has been made in terms of MGARCH models, but there is no statistical theory covering all of them and our results indicate there are differences between conclusions of different forecasting methods.

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