The estimation of a VECM for a small open economy with exogenous variables

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Abstract

One of the most popular ways to model macro economic variables is by the Vector Error Correction Model (VECM). The main reasons for this are that it allows for the possibility to incorporate economic theory through an error correction part, while simultaneously being able to parsimoniously model the relations between the variables over time. Besides forecasting and testing of hypotheses, the VECM is often used for calculating impulse responses, which describe how shocks today affect the variables in the future. In economic theory, a small open economy denotes the economy of a country which is too small to influence the surrounding world. The surrounding world can, for this reason, be seen as exogenous relative to the economy of this small open economy. The main contribution of this paper is the proposal of how to estimate a VECM with exogeneity restrictions on the short run dynamics between small open economies and the surrounding world. A Monte Carlo simulation of impulse responses shows that the proposed model is considerably more efficient, in mean squared error sense, compared to ignoring exogeneity. It is also shown that the empirical size when testing for the number of long run relations is closer to the nominal size. Using empirical data, the small open economy of Sweden is used to exemplify the conclusions of the Monte Carlo simulation.

Keywords: VECM; Impulse responses; Small open economy; Exogeneity.

1. Introduction

A small open economy is an economy which is too small to influence the surrounding world, e.g. in terms of world prices or interest rates. This is true for most countries today, except for the leading economies such as the US and China, or regions like the EU in which the countries grouped together no longer constitute a small open economy. Hence, there is a great interest in developing economic theories that apply to small open economies. Statistics has played a crucial role in economics since the seminal paper of Haavelmo (1944), which lay the statistical foundation for applied macro economics and theoretically justified the work of the Cowles commission. The main focus when modeling macro economic relations was, at the time, on large systems of equations and the majority of the work in macro econometrics focused on related issues, including identification, endogeneity, system estimation, etc. Focus was shifted when Sims (1980) criticized large scale econometric models and popularized the VAR approach. An important contribution of the Sims (1980) paper is that he showed how useful impulse responses can be.

Engle and Granger (1987) introduced cointegration with the possibility of modeling economic equilibriums. The main breakthrough has been the Johansen approach to cointegration (see e.g. Johansen, 1988, 1991, 1995) which merged the VAR model with cointegration. This made it possible to test economic theories through the cointegrating relations and stochastic trends, but also for policy evaluations using impulse responses. With the Johansen approach it is possible to estimate and test restrictions on the cointegrating, or equilibrium, relations, as well as on the adjustment parameters, which describe how the system moves towards equilibrium. The interpretation of a restriction on the adjustment parameters is that the feedback from deviations from the long run equilibrium, i.e. the cointegrating relation, to the corresponding dependent variable is constrained. This is the concept of weak exogeneity applied to cointegration, see e.g. Engle, Hendry and Richard (1983). Johansen proposed to use reduced rank regression to estimate the parameters of the cointegrating relations and the adjustment parameters. To solve the problem with short run dynamics, the first step is to use the Frisch-Waugh-Lovell theorem to concentrate out the short run dynamics. When relating the model to an small open economy it is customary to have two sets of variables. The first set is the variables of interest for

the economy of that country (e.g. GDP, exports, imports and inflation) and the other set contains foreign variables (such as foreign GDP and interest rates). It is important to notice that the concept of a small open economy implies no feedback from the small economy to the foreign, as these can be seen as price takers. Currently, if this is taken into account in the model it is usually accomplished by restricting the appropriate adjustment parameters to zero, but oftentimes it is simply ignored. In possibly restricting the short run parameters as well, a problem arises as they cannot be concentrated out as in Johansen's approach. Hence, restrictions on the short run parameters are very rare, but can be made using the results of Lütkepohl (2005) implemented in the software jMulti. The drawback is that it is not possible to simultaneously have restrictions on the short run dynamics and the adjustment parameters. The purpose of this paper is to propose an estimation procedure which can accommodate restrictions on the short run dynamics, the adjustment parameters and the cointegrating relations simultaneously. The procedure is based on the results of Boswijk (1995) and Groen and Kleibergen (2003).
The paper is organized as follows. The next section introduces the model and the main restrictions of interest as well as the estimation procedure. Section 3 analyzes the performance of imposing the restrictions by Monte Carlo methods while an empirical example is the topic of Section 4. A conclusion ends the paper. In the Appendix a matrix F is defined which we use in Section 2.

2. The model and estimation

The vector error correction model, VECM, for the $k \times 1$ vector y_t can be written, ignoring deterministic terms, as

$$\Delta y_t = \alpha\beta' y_{t-1} + \sum_{i=1}^{p} \Gamma_i \Delta y_{t-1} + \varepsilon_t \tag{1}$$

where α and β are full column rank matrices of size $k \times r$, Γ_i are $k \times k$ matrices containing the short run dynamics parameters, ε_t is a $k \times 1$ vector of white noise disturbances with covariance matrix Ω and $\Delta y_t = y_t - y_{t-1}$. Given some conditions, see e.g. Johansen (1995), we have that y_t is integrated of order one, i.e. it is dominated by a random walk, its first difference Δy_t is stationary and the cointegrating relation $\beta' y_t$ is stationary (and can be thought of as economic equilibriums). The matrix α describes how quick deviations from the cointegrating relations vanish.
Next, let $y_t' = \left[y_t^{exo\prime}, y_t^{end\prime}\right]'$ where y_t^{exo} is the set of s (exogenous) variables from the foreign economy and y_t^{end} the $k-s$ (endogenous) domestic variables. Then

$$\begin{aligned} \begin{bmatrix} \Delta y_t^{exo} \\ \Delta y_t^{end} \end{bmatrix} &= \alpha\beta' \begin{bmatrix} y_{t-1}^{exo} \\ y_{t-1}^{end} \end{bmatrix} + \sum_{i=1}^{p} \Gamma_i \begin{bmatrix} \Delta y_{t-i}^{exo} \\ \Delta y_{t-i}^{end} \end{bmatrix} + \varepsilon_t \\ &= \begin{bmatrix} 0 \\ \alpha^{end} \end{bmatrix} \beta' \begin{bmatrix} y_{t-1}^{exo} \\ y_{t-1}^{end} \end{bmatrix} + \sum_{i=1}^{p} \begin{bmatrix} \Gamma_{11i} & 0 \\ \Gamma_{21i} & \Gamma_{22i} \end{bmatrix} \begin{bmatrix} \Delta y_{t-i}^{exo} \\ \Delta y_{t-i}^{end} \end{bmatrix} + \varepsilon_t. \end{aligned}$$

To estimate the model, we first note that as Δy_{t-i}^{exo} influence all left hand variables and are without restrictions we can use the Frisch-Waugh-Lovell theorem to concentrate them out. Strictly speaking, this means we should replace y_t^{exo} with $\hat{\varepsilon}_t^{exo}$ etc, i.e. the residuals are from this first-step regression, but for simplicity we do not do that. Next, stack the observations which yields the model in matrix form

$$Y = Y_{-1}\Pi + \varepsilon$$

where

$$\Pi = \begin{bmatrix} \multicolumn{2}{c}{\beta\alpha'} \\ 0' & \Gamma_{221}' \\ \vdots & \vdots \\ 0' & \Gamma_{22p}' \end{bmatrix}$$

and

$$Y = \begin{bmatrix} \Delta y_1' \\ \Delta y_2' \\ \vdots \\ \Delta y_T' \end{bmatrix}, Y_{-1} = \begin{bmatrix} y_0' & \Delta y_{-1}^{end\prime} & \cdots & \Delta y_{-p}^{end\prime} \\ y_1' & \Delta y_0^{end\prime} & & \Delta y_{-p+1}^{end\prime} \\ \vdots & & \ddots & \\ y_{T-1}' & \Delta y_{T-1}^{end\prime} & & \Delta y_{T-p}^{end\prime} \end{bmatrix}$$

It can be seen that the GMM objective function is

$$G(\Pi, \Omega) = vec\left(Y_{-1}'\left(Y - Y_{-1}\Pi\right)\right)'\left(\Omega \otimes \left(Y_{-1}'Y_{-1}\right)\right)^{-1} vec\left(Y_{-1}'\left(Y - Y_{-1}\Pi\right)\right)$$

To minimize this we first need to rewrite $vec\left(Y_{-1}'\left(Y - Y_{-1}\Pi\right)\right)$ as

$$\begin{aligned} vec\left(Y_{-1}'\left(Y - Y_{-1}\Pi\right)\right) &= vec\left(Y_{-1}'Y\right) - vec\left(Y_{-1}'Y_{-1}\Pi\right) \\ &= vec\left(Y_{-1}'Y\right) - F_i\Pi_i^{vec} \end{aligned}$$

where

$$\Pi_\beta^{vec} = \begin{bmatrix} vec\left(\beta'\right) \\ vec\left(\Gamma_{221}\right) \\ \vdots \\ vec\left(\Gamma_{22p}\right) \end{bmatrix}, \Pi_\alpha^{vec} = \begin{bmatrix} vec\left(\alpha\right) \\ vec\left(\Gamma_{221}\right) \\ \vdots \\ vec\left(\Gamma_{22p}\right) \end{bmatrix}$$

and F_i, $i = \alpha, \beta$ are suitable matrices which can be found in the Appendix. Conditional on α we have

$$\Pi_\beta^{vec} = \left(F_\beta'\left(\Omega \otimes \left(Y_{-1}'Y_{-1}\right)\right)F_\beta\right)^{-1} F_\beta'\left(\Omega \otimes \left(Y_{-1}'Y_{-1}\right)\right)^{-1} vec\left(Y_{-1}'Y\right)$$

and similarly for Π_α^{vec}. The estimation procedure is:

1. Estimate Ω, α from an unrestricted VECM as these are consistent
2. Estimate β, Γ using F_β, conditional on α and Ω
3. Conditional on α, β, Γ, estimate Ω
4. Estimate α, Γ using F_α, conditional on β and Ω
5. Conditional on α, β, Γ, estimate Ω
6. Iterate 2-5 until convergence

The asymptotic properties of the estimation procedure are the same as in the standard VECM as restrictions on the short run dynamics are asymptotically independent of the long run properties, see e.g. Johansen (2005).

3. Monte Carlo simulations and some results

To analyze the small sample properties we generate data according to (1). We generate five variables, whereof two exogenous, with two cointegrating relations and three lags, i.e. $n = 5$, $s = 2$, $p = 3$ for the sample sizes $T = 100, 125, 150, \ldots, 500$. The parameter values are randomly chosen prior to the simulation and not altered. The number of replicates is 5000. As one main purpose of many macro economic modeling exercises is to estimate impulse responses we mainly evaluate using deviations from the true impulse responses, in form of bias and MSE. The size property of testing the null of two cointegrating relations and the power of testing the null of one relationship is also investigated. The models we compare are the following VECM i) unrestricted, ii) restrictions on α, iii) restrictions on Γ_i, and iv) restrictions on both α and Γ_i.

In Figure 1 some results of the Monte Carlo simulation are displayed. Imposing restrictions yields an empirical size marginally closer to the nominal. The power (i.e. when testing for one cointegrating vector when there are two) is larger without restrictions but this is due to higher empirical size such that the size adjusted power is larger for the restricted case. Concerning impulse responses and Mean Squared Error there is not much difference for the first four periods, but then there are major differences. Imposing restrictions on both adjustment parameters and short run dynamics seems to perform best overall while no restrictions is a worst case scenario. Imposing either restrictions on the adjustment parameters or the short run dynamics performs better than no restrictions and sometimes as good as imposing both. There is no clear winner between imposing one of the restrictions. An interesting result is that when increasing the periods of the impulse responses MSE first increases but then it decreases for the case of both set of restrictions while increase when there are no restrictions. Imposing one of the two restrictions sometimes decreases and sometime do not decrease MSE after the initial increase.

4. Empirical example Ongoing work...

5. Conclusions

In this paper we have proposed the use of an estimation procedure in the case of exogeneity restrictions in a VECM. A Monte Carlo simulation is used to show the advantages of imposing such restrictions. It is found that it is advantageous, in terms of MSE, to impose restrictions on both the short run dynamics as well as on the adjustment parameters. Ignoring restrictions will most often substantially increase MSE. Using one set of restrictions is most often significantly better than no restrictions, but worse than using both. There is no clear winner between restrictions on the short run dynamics or on the adjustment parameters when using only one set of restrictions. The size and power properties are improved, but not greatly.

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Appendix

Let $K_{m,n}$ denote the commutation matrix defined by $K_{m,n} vec\left(A\right) = \left(A'\right)$ then

$$F_{\beta} = \left(I \otimes Y_{-1}' Y_{-1}\right) K_{k+p(k-s),k} \begin{bmatrix} I_{k^2} & 0_{k^2 \times pk(k-s)} \\ 0_{pk(k-s),k^2} & I_p \otimes K_{k,(k-s)} \end{bmatrix} \begin{bmatrix} I_{k^2} & 0_{k^2 \times p(k-s)^2} \\ 0_{pk(k-s) \times k^2} & I_p \otimes \begin{bmatrix} 0_{(k-s)s \times (k-s)^2} \\ I_{(k-s)^2} \end{bmatrix} \end{bmatrix}$$

and

$$F_{\alpha} = \left(I \otimes Y_{-1}' Y_{-1}\right) K_{k,(p+1)k} \begin{bmatrix} K_{k,k} & \begin{matrix} 0_{(k-r)k \times r^2} \\ I_r \otimes \beta \end{matrix} & \begin{matrix} 0_{(k-r)k \times (k-1)kp} \\ 0_{rk \times (k-1)kp} \end{matrix} \\ 0_{pk^2 \times k^2} & & I_{pk} \otimes \begin{bmatrix} I_{k-s} \\ 0_{s \times (k-s)} \end{bmatrix} \end{bmatrix}$$

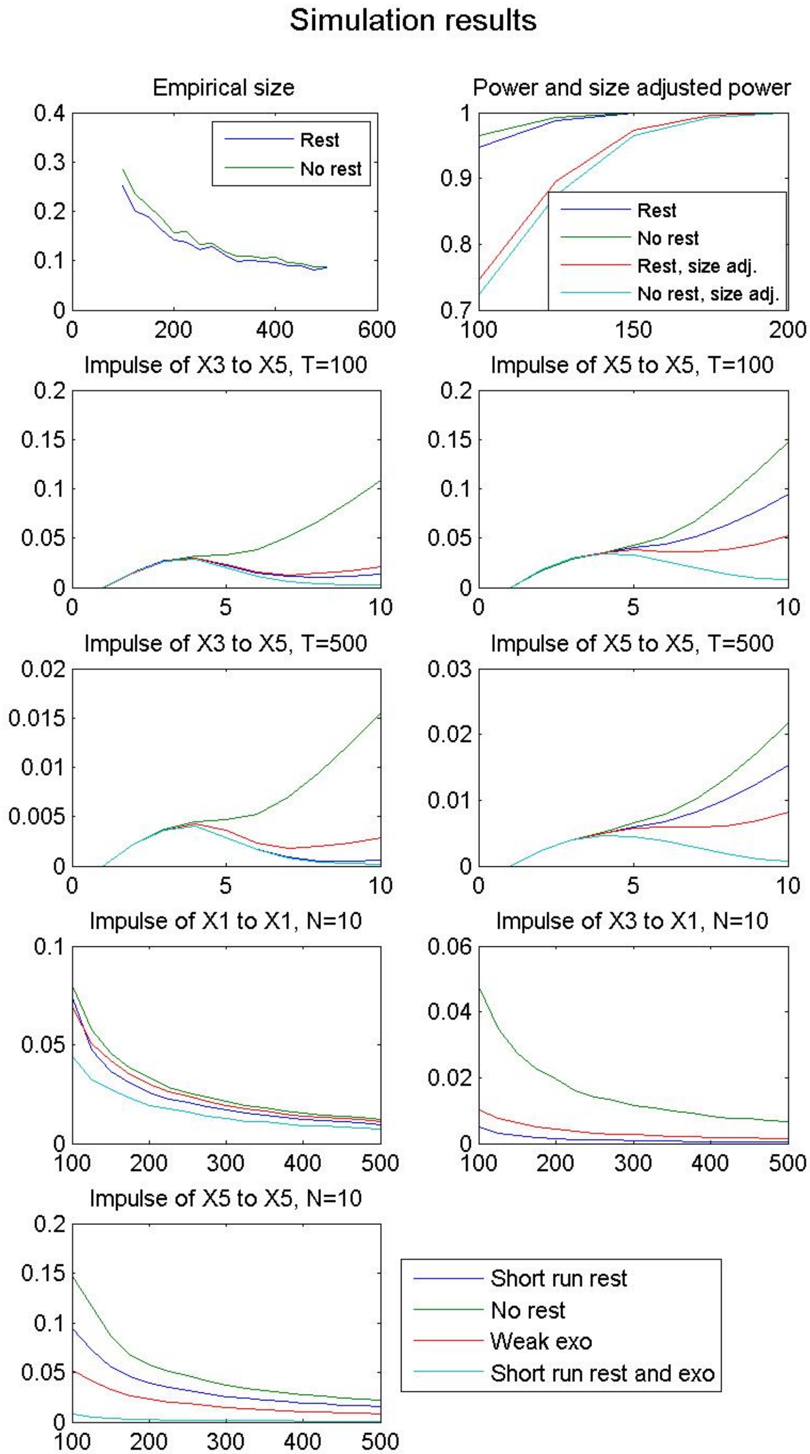

Figure 1: Simulation results as measured by Mean Squared Error. On x-axes there are impulse response periods or sample sizes.