Verification of absolute nonsingularity of 3-tensors through positivity of a single variate characteristic function

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Tensor (multi-way array) data analysis is now a rapidly developing field in statistics, which can analyze more complex data than matrices. Tensor rank is an index of complexity of a datum, and rank determination of tensors is one of fundamental problems in tensor data analysis. Maximal rank and typical rank have attracted many researchers' interest in this field, however, are still not solved completely. Typical rank is important since tensors can not have ranks other than typical ranks with probability 1. Recently authors defined a class of absolutely nonsingular 3-tensors and absolutely full column rank 3-tensors and proved that the existence of an absolutely nonsingular tensor or a full column rank tensor is essential for determining a typical rank of a special size of 3-tensors. Absolute nonsingularity and absolutely full column rank of a 3-tensor are verified by the positivity of their determinant polynomial. There are several algebraic methods to verify the positivity of a multivariate polynomial, for example, Polya's method. In this talk, we discuss about reduction of verification of positivity of a multivariate determinant polynomial to one of positivity of a single variate function, which makes the problem more tractable. We claim the equivalence of Cukierman's characteristic function and Canny's one and hence reduces the calculation of Cukierma's characteristic function to the calculation of a multivariate resultant. Furthermore, we show examples of verification of absolutely nonsingular tensors through Cukierman's theorem. A direction of further researches is also commented.
Keywords: Typical rank; absolutely nonsingular 3-tensors; absolutely full column rank 3-tensors; positivity of determinant polynomials; positivity of characteristic functions.