Mixture of exponentiated Pareto model through Beta type generator

Makgai, S., Bekker, A., Ferreira, J.T., and Arashi, M.
Department of Statistics, University of Pretoria, Pretoria, South Africa
seite.makgai@up.ac.za

Abstract

The beta distribution has been widely used to model a variety of uncertainties as well as probability distributions of variables. The Pareto distribution is known in the modelling and analysis of lifetimes. We propose a flexible model called the generalised beta type III-Pareto distribution, to extend the Kumaraswamy-Pareto distribution, the beta type I-Pareto distribution and several other models. Taking the Pareto as the parent distribution and the beta as the generator, new contributions through weighted cumulative distribution functions can be investigated. The statistical properties of this new model such as the hazard function and moments are derived. The method of maximum likelihood is used to estimate the model parameters and the potential of this newly proposed model is investigated through an application of a real data set.

Keywords: Kumaraswamy; maximum likelihood estimation; parent distribution; Pareto distribution.

1. Introduction

The generator approach proposed by Eugene et al (2002) and Jones (2004) forms the genesis of this new model. The beta generator class (BG) may be characterised by the cumulative distribution function (cdf):

$$F(w) = \int_0^{G(x)} h(w)\, dw$$

where $G(x)$ is termed the parent distribution and $h(w)$ the generator distribution, with corresponding probability density function (pdf)

$$\begin{aligned} f(x) &= \frac{d}{dx} F(x) \\ &= \frac{1}{B(\alpha,\beta)} G(x)^{\alpha-1}\left(1-G(x)\right)^{\beta-1} g(x), \qquad 0 \le x \le 1. \end{aligned}$$

Jones (2012) extended this approach by proposing the Kumaraswamy distribution instead of the beta as the generator. Alexander et al (2012) proposed a flexible generator distribution namely the generalized beta model which includes the classical beta and Kumaraswamy.

In this paper, we combine the model proposed by Cardeño et al (2005) (see also Ehlers et al (2009)) and the work of Alexander et al (2012).

2. Model

Akinsete et al (2008), defined and studied the four parameter beta–Pareto distribution which is well known in literature. In the paper, the Pareto distribution is taken to be the parent distribution with cdf $G(x)$, where

$$G(x) = 1 - \left(\frac{x}{\theta}\right)^{-k}, \qquad x \geq \theta > 0$$

with pdf

$$g(x) = \frac{k\theta^k}{x^{k+1}}, \qquad x \geq \theta,\ k > 0.$$

The generalized distributions are then obtained by taking the parent $G(x)$ distribution in the cdf of a beta distribution with two additional shape parameters, whose role is to introduce skewness and to vary tail weight. Following the same approach with an alternative generator and the Pareto as the parent distribution, this flexible new model is introduced.

3. Application

In this section, this model is fitted to a data set. The data is based on the exceedances of flood peaks (in m^3/s) of the Wheaton River near Carcross in Yukon Territory, Canada (Akinsete et al (2008)). The Akaike information criterion (AIC), Bayesian information criterion (BIC) and the Conditional Akaike information criterion (CAIC) are used for the comparison of the fits. The parameters are estimated by using the method of maximum likelihood.

4. Conclusion

A mixture of exponentiated Pareto model through the beta type III generator is proposed. The various properties of this model is investigated. Properties include studying the density function, hazard function and the moments such as skewness and kurtosis. The model is found to be unimodal and contain the same special cases of existing distributions that are known in literature. The method of maximum likelihood is used to estimate the parameters of the model. In conclusion, proposing a mixture of exponentiated Pareto model provides a wealth of models not yet familiar to statisticians.

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