Pairwise and Other Post-hoc Tests of Spatial Clustering Based on NNCTs

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Abstract

Spatial clustering patterns in a multiple class setting such as segregation and association can be tested using nearest neighbor contingency tables (NNCTs). A NNCT is constructed based on the types and frequencies of the nearest neighbor (NN) pairs. We consider the cell-specific (or pairwise) and overall segregation tests based on NNCTs in literature and introduce new ones and determine their asymptotic distributions. We demonstrate that cell-specific tests enjoy asymptotic normality, while overall tests have chi-square distributions asymptotically. We also perform an extensive Monte Carlo simulation study to compare the finite sample performance of the tests in terms of empirical size and power based on the asymptotic and Monte Carlo critical values and determine the tests that have the best size and power performance and are robust to differences in relative abundances (of the classes). In addition to the cell-specific tests, we discuss one(-class)-versus-rest type of tests as post-hoc tests after a significant overall test. We compare the new tests with the existing NNCT-tests in literature with simulations and illustrate the tests on an ecological data set.

Keywords: Association; asymptotic distribution; completely mapped data; complete spatial randomness; overall tests; random labeling; segregation

1 Introduction

Spatial clustering of two or more classes with respect to each other can have important consequences in various fields. Multi-class clustering patterns such as segregation and association might result from the interaction between two or more classes (or species).
In this article, we survey these NNCT-tests and also introduce new cell-specific segregation tests (i.e., tests for each cell or entry in the NNCT) and the corresponding overall tests for completely mapped data. We demonstrate that cell-specific tests are asymptotically normal, and overall tests tend to chi-square distributions with respective degrees of freedom as class sizes tend to infinity. In practice, cell-specific tests can serve as post-hoc tests to be performed when an overall segregation test yields a significant result. In Ceyhan (2008), cell-specific tests are compared for two types of NNCT-tests. As an alternative post-hoc test after a significant overall test, we discuss one-class-versus-rest (or one-vs-rest) type of NNCT-tests. By extensive Monte Carlo simulations, we compare these tests with the existing NNCT-tests in literature (i.e., with the ones proposed in Dixon (1994, 2002) and Ceyhan (2008, 2010)) in terms of empirical size and power based on asymptotic and Monte Carlo critical values, and thus determine which tests perform better for the segregation or association alternative and which ones are more robust to differences in relative abundances (of the classes). There are also methods to test spatial correlation of lattice data, such as Moran's I (Moran (1950)), Ord's statistics (Ord (1975)) and recently developed approximate profile likelihood estimator (Li et al. (2011)). However, these statistics have been introduced to identify spatial clusters and spatial interactions among areas. But our cell-specific and overall NNCT-tests, though detecting/testing spatial interaction, are not based on area or aggregated data. The term "cell" in our cell-specific test does not refer to areas or quadrats, but to the entries in the NNCT.

2 NNCTs and Null and Alternative Spatial Patterns

NNCTs are constructed using the NN frequencies of classes. In this article, to find the NN of a point, we employ the usual Euclidean distance. We provide a brief description for $m \geq 2$ classes. Suppose there are m classes labeled as $\{1, 2, \dots, m\}$. NNCTs are constructed using NN frequencies for each class. Let N_i be the number of points from class i for $i \in \{1, 2, \dots, m\}$ and $n = \sum_{i=1}^{m} N_i$ and n points be denoted as $\{w_1, w_2, \dots, w_n\}$. If we record the class of each point and its NN, the NN relationships fall into m^2 categories:

$$(1,1),\ (1,2), \dots, (1,m);\ (2,1),\ (2,2), \dots, (2,m); \dots, (m,m)$$

where in category or cell (i, j), class i is called the *base class* and class j is called the *NN class*. Denoting N_{ij} as the observed frequency of category (i, j) for $i, j \in \{1, 2, \dots, m\}$, we obtain the NNCT, $\mathcal{N}$, in Table 1 where C_j is the sum of column j; i.e., number of times class j points serve as NNs for $j \in \{1, 2, \dots, m\}$. Note also that $n = \sum_{i,j} N_{ij}$, $n_i = \sum_{j=1}^{m} N_{ij}$, $C_j = \sum_{i=1}^{m} N_{ij}$, and

$$N_{ij} = \sum_{l=1}^{n} \sum_{k=1}^{n} \mathbf{I}(w_k \text{ is NN of } w_l) \cdot \mathbf{I}(w_l \text{ is from class } i) \cdot \mathbf{I}(w_k \text{ is from class } j),$$

where $w_k \neq w_l$ and $\mathbf{I}(\cdot)$ denotes the indicator function. Here we adopt the convention that variables denoted by upper case letters are random, while variables denoted by lower case letters are fixed. Thus, in our NNCT-analysis, row sums are assumed to be fixed (i.e., class sizes are given), while column sums are assumed to be random and depend on the NN relationships between the classes.

		NN class			
		class 1	...	class m	total
base class	class 1	N_{11}	...	N_{1m}	n_1
	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
	class m	N_{m1}	...	N_{mm}	n_m
	total	C_1	...	C_m	n

Table 1: The NNCT, $\mathcal{N}$, for m classes.

We describe the spatial point patterns for two classes only; the extension to multi-class case is straightforward. Our null hypothesis is

$$H_o : \text{randomness in the NN structure with NN probabilities being proportional to class frequencies}$$

which may result from *random labeling* (RL) or (an appropriate type of) *independence* of points from two classes. Under independence, the two classes result independently from the same stochastic process which ensures their spatial distribution is identical. In this article, among independence patterns we will consider complete spatial randomness (CSR) of points from two classes. Roughly, under *CSR independence*, two classes are independently uniformly distributed in a region of interest, while RL is the pattern in which, given a fixed set of points in a region, class labels are assigned to these fixed points randomly so that the labels are independent of the locations. For CSR independence, we condition on $N_i = n_i$; i.e., we work with a binomial process. If it is desired to have the class sizes N_i to be random, we may consider a spatial Poisson point process on the region of interest as our null hypothesis. The null model in a NNCT analysis depends on the particular ecological context. Goreaud and Pélissier (2003) state that under CSR independence, the two classes are *a priori* the result of different processes (e.g., individuals of different species or age cohorts). On the other hand, under RL, some processes affect *a posteriori* the individuals of a single population (e.g., diseased vs. non-diseased individuals of a single species).

As alternatives, we consider two major types of deviations from H_o, namely, segregation and association. *Segregation* occurs if the NN of an individual is more likely to be of the same class as the individual than to be from a different class. That is, the probability that this individual having a NN from the same class is larger than the relative frequency of the same class (see, e.g., Pielou (1961)). *Association* occurs if the NN of an individual is more likely to be from another class than to be of the same class as the individual. That is, the probability that this individual having a NN from another class is larger than the relative frequency of the other class in question. These patterns are not symmetric, e.g., for two classes, one class might be more associated with the other class. For example, plant species X could be more dependent on species Y, hence X plants occur in close vicinity of Y plants, while the reverse relation may not be in the same level or type. Also, class X points might exhibit a stronger clustering, compared to class Y points, and thus might be more segregated compared to class Y points. See Ceyhan (2010) for more details on the null and alternative patterns.

3 Cell-Specific and Overall Segregation Tests

3.1 Dixon's Cell-Specific and Overall Segregation Tests

Dixon's cell-specific tests are used to measure the deviation of observed count in cell (i, j) in a NNCT from its expected value under H_o described in detail in, e.g., Dixon (1994, 2002). Here, we provide the details for completeness. The test statistic suggested by Dixon for cell (i, j) is given by

$$Z^D_{ij} = \frac{N_{ij} - \mathbf{E}[N_{ij}]}{\sqrt{\mathbf{Var}[N_{ij}]}}, \tag{1}$$

where $\mathbf{E}[N_{ij}]$ is the expected cell count and $\mathbf{Var}[N_{ij}]$ is the variance of cell count N_{ij}.
For $m \geq 2$ classes, under RL or CSR independence, the expected cell count for cell (i, j) is

$$\mathbf{E}[N_{ij}] = \begin{cases} n_i(n_i - 1)/(n - 1) & \text{if } i = j, \\ n_i\, n_j/(n - 1) & \text{if } i \neq j, \end{cases} \tag{2}$$

where n_i is the fixed sample size for class i for $i = 1, 2, \ldots, m$. Observe that the expected cell counts depend only on the class sizes (i.e., row sums), but not on the column sums. And the variance is

$$\mathbf{Var}[N_{ij}] = \begin{cases} (n + R)\, p_{ii} + (2\,n - 2\,R + Q)\, p_{iii} + (n^2 - 3\,n - Q + R)\, p_{iiii} - (n\, p_{ii})^2 & \text{if } i = j, \\ n\, p_{ij} + Q\, p_{iij} + (n^2 - 3\,n - Q + R)\, p_{iijj} - (n\, p_{ij})^2 & \text{if } i \neq j, \end{cases} \tag{3}$$

with p_{xx}, p_{xxx}, and p_{xxxx} are the probabilities that a randomly picked pair, triplet, or quartet of points, respectively, are the indicated classes and are given by

$$\begin{aligned} p_{ii} &= \frac{n_i\,(n_i - 1)}{n\,(n - 1)}, & p_{ij} &= \frac{n_i\, n_j}{n\,(n - 1)}, \\ p_{iii} &= \frac{n_i\,(n_i - 1)\,(n_i - 2)}{n\,(n - 1)\,(n - 2)}, & p_{iij} &= \frac{n_i\,(n_i - 1)\, n_j}{n\,(n - 1)\,(n - 2)}, \\ p_{iiii} &= \frac{n_i\,(n_i - 1)\,(n_i - 2)\,(n_i - 3)}{n\,(n - 1)\,(n - 2)\,(n - 3)}, & p_{iijj} &= \frac{n_i\,(n_i - 1)\, n_j\,(n_j - 1)}{n\,(n - 1)\,(n - 2)\,(n - 3)}. \end{aligned} \tag{4}$$

Furthermore, R is twice the number of reflexive pairs and Q is the number of points with shared NNs, which occurs when two or more points share a NN. Then $Q = 2\,(Q_2 + 3\,Q_3 + 6\,Q_4 + 10\,Q_5 + 15\,Q_6)$ where Q_j is the number of points that serve as a NN to other points j times.
In the multi-class case with m classes, combining the m^2 cell-specific tests in Section 3.1, Dixon (2002) suggests the following quadratic form to obtain the overall segregation test:

$$\mathcal{X}_D = (\mathbf{N} - \mathbf{E}[\mathbf{N}])' \Sigma^-_D (\mathbf{N} - \mathbf{E}[\mathbf{N}]) \tag{5}$$

where $\mathbf{N}$ is the $m^2 \times 1$ vector of m rows of the NNCT concatenated row-wise, $\mathbf{E}[\mathbf{N}]$ is the vector of $\mathbf{E}[N_{ij}]$ which are as in Equation (2), Σ_D is the $m^2 \times m^2$ variance-covariance matrix for the cell count vector $\mathbf{N}$ with diagonal entries being equal to $\mathbf{Var}[N_{ij}]$ and off-diagonal entries being $\mathbf{Cov}[N_{ij}, N_{kl}]$ for $(i, j) \neq (k, l)$. The explicit forms of the variance and covariance terms are provided in Dixon (2002). Also, Σ^-_D is a generalized inverse of Σ_D (Searle (2006)) and $'$ stands for the transpose of a vector or matrix. Then under RL, $\mathcal{X}_D$ has a $\chi^2_{m(m-1)}$ distribution asymptotically.

3.2 Type I Cell-Specific and Overall Segregation Tests

In standard cases like multinomial sampling for contingency tables with fixed row totals and conditioning on the column totals, $C_j = c_j$, the expected cell count for cell (i,j) in contingency tables is $\mathbf{E}[N_{ij}] = \frac{n_i\, c_j}{n}$. We first consider the difference $N_{ij} - \frac{n_i\, c_j}{n}$ for cell (i,j). However under RL, $N_i = n_i$ are fixed, but C_j are random quantities and $C_j = \sum_{i=1}^m N_{ij}$, hence we suggest as the first type of cell-specific segregation test as

$$T^I_{ij} = N_{ij} - \frac{n_i\, C_j}{n}.$$

Then under RL,

$$\mathbf{E}\left[T^I_{ij}\right] = \begin{cases} \frac{n_i(n_i-1)}{(n-1)} - \frac{n_i}{n}\,\mathbf{E}[C_i] & \text{if } i = j,\\ \frac{n_i\, n_j}{(n-1)} - \frac{n_i}{n}\,\mathbf{E}[C_j] & \text{if } i \neq j. \end{cases} \tag{6}$$

For all j, $\mathbf{E}[C_j] = n_j$, since

$$\begin{aligned}\mathbf{E}[C_j] &= \sum_{i=1}^m \mathbf{E}[N_{ij}] = \frac{n_j(n_j-1)}{(n-1)} + \sum_{i\neq j}\frac{n_i n_j}{(n-1)} = \frac{n_j(n_j-1)}{(n-1)} + \frac{n_j}{(n-1)}\sum_{i\neq j} n_i\\ &= \frac{n_j(n_j-1)}{(n-1)} + \frac{n_j}{(n-1)}(n-n_j) = n_j.\end{aligned}$$

Therefore,

$$\mathbf{E}\left[T^I_{ij}\right] = \begin{cases} \frac{n_i(n_i-n)}{n(n-1)} & \text{if } i = j,\\ \frac{n_i\, n_j}{n(n-1)} & \text{if } i \neq j. \end{cases} \tag{7}$$

For the variance of T^I_{ij}, we have

$$\mathbf{Var}\left[T^I_{ij}\right] = \mathbf{Var}[N_{ij}] + \left(\frac{n_i^2}{n^2}\right)\mathbf{Var}[C_j] - 2\left(\frac{n_i}{n}\right)\mathbf{Cov}[N_{ij}, C_j] \tag{8}$$

where $\mathbf{Var}[N_{ij}]$ are as in Equation (3), $\mathbf{Var}[C_j] = \sum_{i=1}^m \mathbf{Var}[N_{ij}] + \sum_{k\neq i}\sum_i \mathbf{Cov}[N_{ij}, N_{kj}]$ and $\mathbf{Cov}[N_{ij}, C_j] = \sum_{k=1}^m \mathbf{Cov}[N_{ij}, N_{kj}]$ with $\mathbf{Cov}[N_{ij}, N_{kl}]$ are as in Equations (4)-(12) of Dixon (2002).
As a new cell-specific test, we propose

$$Z^I_{ij} = \frac{T^I_{ij} - \mathbf{E}\left[T^I_{ij}\right]}{\sqrt{\mathbf{Var}\left[T^I_{ij}\right]}}. \tag{9}$$

We can also combine the type I cell-specific tests of Section 3.2. Let $\mathbf{T_I}$ be the vector of m^2 T^I_{ij} values, i.e.,

$$\mathbf{T_I} = \left(T^I_{11}, T^I_{12}, \ldots, T^I_{1m}, T^I_{21}, T^I_{22}, \ldots, T^I_{2m}, \ldots, T^I_{mm}\right)',$$

and let $\mathbf{E}\left[\mathbf{T_I}\right]$ be the vector of $\mathbf{E}\left[T^I_{ij}\right]$ values. Note that

$$\mathbf{E}\left[\mathbf{T_I}'\right] = \Big(\mathbf{E}\left[T^I_{11}\right], \mathbf{E}\left[T^I_{12}\right], \ldots, \mathbf{E}\left[T^I_{1m}\right], \mathbf{E}\left[T^I_{21}\right], \mathbf{E}\left[T^I_{22}\right], \ldots, \mathbf{E}\left[T^I_{2m}\right], \ldots, \mathbf{E}\left[T^I_{mm}\right]\Big).$$

Hence to obtain a new overall segregation test, referred to as *type I overall test*, we use the following quadratic form:

$$\mathcal{X}_I = \left(\mathbf{T_I} - \mathbf{E}\left[\mathbf{T_I}\right]\right)' \Sigma_I^- \left(\mathbf{T_I} - \mathbf{E}\left[\mathbf{T_I}\right]\right) \tag{10}$$

where Σ_I is the $m^2 \times m^2$ variance-covariance matrix of $\mathbf{T_I}$.
Under RL, the diagonal entries in the variance-covariance matrix Σ_I are $\mathbf{Var}\left[T^I_{ij}\right]$ which are provided in Equation (8). For the off-diagonal entries in Σ_I, i.e., $\mathbf{Cov}\left[T^I_{ij}, T^I_{kl}\right]$ with $(i,j) \neq (k,l)$, we have

$$\begin{aligned}\mathbf{Cov}\left[T^I_{ij}, T^I_{kl}\right] &= \mathbf{Cov}\left[N_{ij} - \frac{n_i}{n}C_j, N_{kl} - \frac{n_k}{n}C_l\right] =\\ &\mathbf{Cov}[N_{ij}, N_{kl}] - \frac{n_k}{n}\mathbf{Cov}[N_{ij}, C_l] - \frac{n_i}{n}\mathbf{Cov}[N_{kl}, C_j] + \frac{n_i n_k}{n^2}\mathbf{Cov}[C_j, C_l].\end{aligned}$$

3.3 Type II Cell-Specific and Overall Segregation Tests

In Section 3.2, we suggested $N_{ij} - \frac{n_i\,C_j}{n}$ as the test statistic for cell (i,j). However, under RL, $\mathbf{E}[C_j] = n_j$, so we suggest as the second type of segregation test as

$$T^{II}_{ij} = N_{ij} - \frac{n_i\,n_j}{n}.$$

Then under RL, $\mathbf{E}\left[T^{II}_{ij}\right] = \mathbf{E}\left[T^{I}_{ij}\right]$ which is provided in Equation (7). Moreover, the variance of T^{II}_{ij} is $\mathbf{Var}\left[T^{II}_{ij}\right] = \mathbf{Var}[N_{ij}]$, since n_i, n_j and n are fixed.
As a cell-specific test, we propose

$$Z^{II}_{ij} = \frac{T^{II}_{ij} - \mathbf{E}\left[T^{II}_{ij}\right]}{\sqrt{\mathbf{Var}\left[T^{II}_{ij}\right]}}. \tag{11}$$

We also combine the type II cell-specific tests of Section 3.3. Let $\mathbf{T_{II}}$ be the vector of m^2 T^{II}_{ij} values, i.e.,

$$\mathbf{T_{II}} = \left(T^{II}_{11}, T^{II}_{12}, \ldots, T^{II}_{1m}, T^{II}_{21}, T^{II}_{22}, \ldots, T^{II}_{2m}, \ldots, T^{II}_{mm}\right)',$$

and let $\mathbf{E}\left[\mathbf{T_{II}}\right]$ be the vector of $\mathbf{E}\left[T^{II}_{ij}\right]$ values. As the type II overall segregation test, we use the following quadratic form:

$$\mathcal{X}_{II} = \left(\mathbf{T_{II}} - \mathbf{E}\left[\mathbf{T_{II}}\right]\right)' \Sigma^{-}_{II} \left(\mathbf{T_{II}} - \mathbf{E}\left[\mathbf{T_{II}}\right]\right) \tag{12}$$

where Σ_{II} is the $m^2 \times m^2$ variance-covariance matrix of $\mathbf{T_{II}}$.
Under RL, the diagonal entries in the variance-covariance matrix Σ_N are $\mathbf{Var}\left[T^{II}_{ij}\right]$ which are same as $\mathbf{Var}[N_{ij}]$. For the off-diagonal entries in Σ_{II}, i.e., $\mathbf{Cov}\left[T^{II}_{ij}, T^{II}_{kl}\right]$ with $(i,j) \neq (k,l)$, we have $\mathbf{Cov}\left[T^{II}_{ij}, T^{II}_{kl}\right] = \mathbf{Cov}[N_{ij} - \frac{n_i n_j}{n}, N_{kl} - \frac{n_k n_l}{n}] = \mathbf{Cov}[N_{ij}, N_{kl}]$.

3.4 Type III Cell-Specific and Overall Segregation Tests

In the previous sections, $\mathbf{E}\left[T^{I}_{ij}\right] = \mathbf{E}\left[T^{II}_{ij}\right] \neq 0$ under RL. Hence, instead of these test statistics, in order to have the expected value of our test statistic to be zero, we suggest the following test statistic:

$$T^{III}_{ij} = \begin{cases} N_{ii} - \frac{(n_i-1)}{(n-1)} C_i & \text{if } i = j, \\ N_{ij} - \frac{n_i}{(n-1)} C_j & \text{if } i \neq j. \end{cases} \tag{13}$$

This test statistic is the same as the new cell-specific test introduced in Ceyhan (2010) and details of this test are provided here for completeness. Then $\mathbf{E}\left[T^{III}_{ij}\right] = 0$, since, for $i = j$,

$$\mathbf{E}\left[T^{III}_{ii}\right] = \mathbf{E}[N_{ii}] - \frac{(n_i-1)}{(n-1)}\mathbf{E}[C_i] = \frac{n_i(n_i-1)}{(n-1)} - \frac{(n_i-1)}{(n-1)} n_i = 0,$$

and for $i \neq j$,

$$\mathbf{E}\left[T^{III}_{ij}\right] = \mathbf{E}[N_{ij}] - \frac{(n_i-1)}{(n-1)}\mathbf{E}[C_j] = \frac{n_i\,n_j}{(n-1)} - \frac{(n_i-1)}{(n-1)} n_j = 0.$$

As for the variance of T^{III}_{ij}, we have

$$\mathbf{Var}\left[T^{III}_{ij}\right] = \begin{cases} \mathbf{Var}[N_{ii}] + \frac{(n_i-1)^2}{(n-1)^2}\mathbf{Var}[C_i] - 2\frac{(n_i-1)}{(n-1)}\mathbf{Cov}[N_{ii}, C_i] & \text{if } i = j, \\ \mathbf{Var}[N_{ij}] + \frac{n_i^2}{(n-1)^2}\mathbf{Var}[C_j] - 2\frac{n_i}{(n-1)}\mathbf{Cov}[N_{ij}, C_j] & \text{if } i \neq j. \end{cases} \tag{14}$$

As a new cell-specific test, we propose

$$Z^{III}_{ij} = \frac{T^{III}_{ij}}{\sqrt{\mathbf{Var}\left[T^{III}_{ij}\right]}}. \tag{15}$$

When we combine the type III cell-specific tests of Section 3.4, we obtain type III overall test as follows. Let $\mathbf{T_{III}}$ be the vector of m^2 T_{ij}^{III} values, i.e.,

$$\mathbf{T_{III}} = \left(T_{11}^{III}, T_{12}^{III}, \ldots, T_{1m}^{III}, T_{21}^{III}, T_{22}^{III}, \ldots, T_{2m}^{III}, \ldots, T_{mm}^{III}\right)',$$

and let $\mathbf{E}\left[\mathbf{T_{III}}\right]$ be the vector of $\mathbf{E}\left[T_{ij}^{III}\right]$ values. Note that $\mathbf{E}\left[\mathbf{T_{III}}\right] = \mathbf{0}$ where $\mathbf{0}$ stands for a vector of zeros. As the type III overall segregation test, we use the following quadratic form:

$$\mathcal{X}_{III} = \mathbf{T}'_{\mathbf{III}} \Sigma_{III}^{-} \mathbf{T_{III}} \tag{16}$$

where Σ_{III} is the $m^2 \times m^2$ variance-covariance matrix of $\mathbf{T_{III}}$.
Under RL, the diagonal entries in the variance-covariance matrix Σ_{III} are $\mathbf{Var}\left[T_{ij}^{III}\right]$ which are provided in Equation (14). For the off-diagonal entries in Σ_{III}, i.e., $\mathbf{Cov}\left[T_{ij}^{III}, T_{kl}^{III}\right]$ with $(i,j) \neq (k,l)$, there are four cases to consider:
case 1: $i = j$ and $k = l$, then

$$\mathbf{Cov}\left[T_{ii}^{III}, T_{kk}^{III}\right] = \mathbf{Cov}\left[N_{ii} - \frac{(n_i-1)}{(n-1)}C_i, N_{kk} - \frac{(n_k-1)}{(n-1)}C_k\right] =$$
$$\mathbf{Cov}[N_{ii}, N_{kk}] - \frac{(n_k-1)}{(n-1)}\mathbf{Cov}[N_{ii}, C_k] - \frac{(n_i-1)}{(n-1)}\mathbf{Cov}[N_{kk}, C_i] + \frac{(n_i-1)(n_k-1)}{(n-1)^2}\mathbf{Cov}[C_i, C_k].$$

case 2: $i = j$ and $k \neq l$, then

$$\mathbf{Cov}\left[T_{ii}^{III}, T_{kl}^{III}\right] = \mathbf{Cov}\left[N_{ii} - \frac{(n_i-1)}{(n-1)}C_i, N_{kl} - \frac{n_k}{(n-1)}C_l\right] =$$
$$\mathbf{Cov}[N_{ii}, N_{kl}] - \frac{n_k}{(n-1)}\mathbf{Cov}[N_{ii}, C_l] - \frac{(n_i-1)}{(n-1)}\mathbf{Cov}[N_{kl}, C_i] + \frac{(n_i-1)n_k}{(n-1)^2}\mathbf{Cov}[C_i, C_l].$$

case 3: $i \neq j$ and $k = l$, then $\mathbf{Cov}\left[T_{ij}^{III}, T_{kk}^{III}\right] = \mathbf{Cov}\left[T_{kk}^{III}, T_{ij}^{III}\right]$, which is essentially **case 2** above.
case 4: $i \neq j$ and $k \neq l$, then

$$\mathbf{Cov}\left[T_{ij}^{III}, T_{kl}^{III}\right] = \mathbf{Cov}\left[N_{ij} - \frac{n_i}{(n-1)}C_j, N_{kl} - \frac{n_k}{(n-1)}C_l\right] =$$
$$\mathbf{Cov}[N_{ij}, N_{kl}] - \frac{n_k}{(n-1)}\mathbf{Cov}[N_{ij}, C_l] - \frac{n_i}{(n-1)}\mathbf{Cov}[N_{kl}, C_j] + \frac{n_i n_k}{(n-1)^2}\mathbf{Cov}[C_j, C_l].$$

3.5 Type IV Cell-Specific and Overall Segregation Tests

For T_{ij}^{III}, we introduced a coefficient in front of the second term, i.e., $n_i\, C_j/n$, to obtain the expected value for our statistic to be zero under RL. In this section, we modify the first term and obtain the following test statistic:

$$T_{ij}^{IV} = \begin{cases} \frac{n_i(n-1)}{n(n_i-1)}N_{ii} - \frac{n_i}{n}C_i = \frac{n_i}{n}\left(\frac{n-1}{n_i-1}N_{ii} - C_i\right) & \text{if } i = j, \\ \frac{n-1}{n}N_{ij} - \frac{n_i}{n}C_j = \frac{1}{n}\left((n-1)\, N_{ij} - n_i\, C_j\right) & \text{if } i \neq j. \end{cases} \tag{17}$$

Then $\mathbf{E}\left[T_{ij}^{IV}\right] = 0$, since, for $i = j$,

$$\mathbf{E}\left[T_{ii}^{IV}\right] = \frac{n_i}{n}\left(\frac{n-1}{n_i-1}\mathbf{E}[N_{ii}] - \mathbf{E}[C_i]\right) = \frac{n_i}{n}\left(\frac{n-1}{n_i-1}\frac{n_i(n_i-1)}{n-1} - n_i\right) = 0,$$

and for $i \neq j$,

$$\mathbf{E}\left[T_{ij}^{IV}\right] = \frac{1}{n}\left((n-1)\mathbf{E}[N_{ij}] - n_i\mathbf{E}[C_j]\right) = \frac{1}{n}\left((n-1)\frac{n_i\, n_j}{n-1} - n_i\, n_j\right) = 0.$$

As for the variance of T^{IV}_{ij}, we have

$$\mathbf{Var}\left[T^{IV}_{ij}\right] = \begin{cases} \frac{n_i^2}{n^2}\left(\frac{(n-1)^2}{(n_i-1)^2}\mathbf{Var}[N_{ii}] + \mathbf{Var}[C_i] - 2\frac{(n-1)}{(n_i-1)}\mathbf{Cov}[N_{ii}, C_i]\right) & \text{if } i = j, \\ \frac{1}{n^2}\left((n-1)^2\mathbf{Var}[N_{ij}] + n_i^2\mathbf{Var}[C_j] - 2(n-1)n_i\mathbf{Cov}[N_{ij}, C_j]\right) & \text{if } i \neq j. \end{cases} \tag{18}$$

As a new cell-specific test, we propose

$$Z^{IV}_{ij} = \frac{T^{IV}_{ij}}{\sqrt{\mathbf{Var}\left[T^{IV}_{ij}\right]}}. \tag{19}$$

When we combine the type IV cell-specific tests of Section 3.5, we obtain type IV overall test as follows. Let $\mathbf{T_{IV}}$ be the vector of m^2 T^{IV}_{ij} values, i.e.,

$$\mathbf{T_{IV}} = \left(T^{IV}_{11}, T^{IV}_{12}, \ldots, T^{IV}_{1m}, T^{IV}_{21}, T^{IV}_{22}, \ldots, T^{IV}_{2m}, \ldots, T^{IV}_{mm}\right)',$$

and let $\mathbf{E}\left[\mathbf{T_{IV}}\right]$ be the vector of $\mathbf{E}\left[T^{IV}_{ij}\right]$ values. Note that $\mathbf{E}\left[\mathbf{T_{IV}}\right] = \mathbf{0}$. As the type IV overall segregation test, we use the following quadratic form:

$$\mathcal{X}_{IV} = \mathbf{T}'_{\mathbf{IV}}\Sigma^{-}_{IV}\mathbf{T_{IV}} \tag{20}$$

where Σ_{IV} is the $m^2 \times m^2$ variance-covariance matrix of $\mathbf{T_{IV}}$.
Under RL, the diagonal entries in the variance-covariance matrix Σ_{IV} are $\mathbf{Var}\left[T^{IV}_{ij}\right]$ which are provided in Equation (18). For the off-diagonal entries in Σ_{IV}, i.e., $\mathbf{Cov}\left[T^{IV}_{ij}, T^{IV}_{kl}\right]$ with $(i, j) \neq (k, l)$, there are four cases to consider:
case 1: $i = j$ and $k = l$, then

$$\mathbf{Cov}\left[T^{IV}_{ii}, T^{IV}_{kk}\right] = \mathbf{Cov}\left[\frac{n_i}{n}\left(\frac{n-1}{n_i-1}N_{ii} - C_i\right), \frac{n_k}{n}\left(\frac{n-1}{n_k-1}N_{kk} - \frac{(n_k-1)}{(n-1)}C_k\right)\right] =$$
$$\frac{n_i n_k}{n^2}\mathbf{Cov}\left[\frac{n-1}{n_i-1}N_{ii} - C_i, \frac{n-1}{n_k-1}N_{kk} - \frac{(n_k-1)}{(n-1)}C_k\right] =$$
$$\frac{n_i n_k}{n^2}\left(\frac{(n-1)^2}{(n_i-1)(n_k-1)}\mathbf{Cov}[N_{ii}, N_{kk}] - \frac{(n-1)}{(n_i-1)}\mathbf{Cov}[N_{ii}, C_k] - \frac{(n-1)}{(n_k-1)}\mathbf{Cov}[N_{kk}, C_i] + \mathbf{Cov}[C_i, C_k]\right).$$

case 2: $i = j$ and $k \neq l$, then

$$\mathbf{Cov}\left[T^{IV}_{ii}, T^{IV}_{kl}\right] =$$
$$\mathbf{Cov}\left[\frac{n_i}{n}\left(\frac{n-1}{n_i-1}N_{ii} - C_i\right), \frac{1}{n}\left((n-1)N_{kl} - n_k C_l\right)\right] = \frac{n_i}{n^2}\mathbf{Cov}\left[\frac{n-1}{n_i-1}N_{ii} - C_i, (n-1)N_{kl} - n_k C_l\right] =$$
$$\frac{n_i}{n^2}\left(\frac{(n-1)^2}{(n_i-1)}\mathbf{Cov}[N_{ii}, N_{kl}] - \frac{(n-1)n_k}{(n_i-1)}\mathbf{Cov}[N_{ii}, C_l] - (n-1)\mathbf{Cov}[N_{kl}, C_i] + n_k\mathbf{Cov}[C_i, C_l]\right).$$

case 3: $i \neq j$ and $k = l$, then $\mathbf{Cov}\left[T^{IV}_{ij}, T^{IV}_{kk}\right] = \mathbf{Cov}[T^{IV}_{kk}, T^{IV}_{ij}]$, which is essentially **case 2** above.
case 4: $i \neq j$ and $k \neq l$, then

$$\mathbf{Cov}\left[T^{IV}_{ij}, T^{IV}_{kl}\right] = \mathbf{Cov}\left[\frac{1}{n}\left((n-1)N_{ij} - n_i C_j\right), \frac{1}{n}\left((n-1)N_{kl} - n_k C_l\right)\right] =$$
$$\frac{1}{n^2}\mathbf{Cov}\left[(n-1)N_{ij} - n_i C_j, (n-1)N_{kl} - n_k C_l\right] =$$
$$\frac{1}{n^2}\left((n-1)^2\mathbf{Cov}[N_{ij}, N_{kl}] - (n-1)n_k\mathbf{Cov}[N_{ij}, C_l] - (n-1)n_i\mathbf{Cov}[N_{kl}, C_j] + n_i n_k\mathbf{Cov}[C_j, C_l]\right).$$

In all the above cases, $\mathbf{Cov}[N_{ij}, N_{kl}]$ are as in Dixon (2002), $\mathbf{Cov}[N_{ij}, C_l] = \sum_{k=1}^{m}\mathbf{Cov}[N_{ij}, N_{kl}]$ and $\mathbf{Cov}[C_i, C_j] = \sum_{k=1}^{m}\sum_{l=1}^{m}\mathbf{Cov}[N_{ki}, N_{lj}]$.

3.6 Post-hoc Tests after the Overall Tests: Class-Specific, Pairwise, and One-vs-rest Type Tests

In our construction of the NNCT-tests, although we first introduce the cell-specific tests and then develop overall tests based on the cell-specific tests, in practice, it is more natural to conduct the tests in reverse order. That is, first an overall NNCT-test could be performed, and if significant, then one can perform cell-specific tests to determine the types and levels of the spatial interaction patterns between the classes. This procedure is somewhat analogous to ANOVA F-test to compare multiple groups, in the sense that if the F-test yields a significant result, then one performs pairwise tests to determine which pairs are different. To this end, one can conduct several post-hoc tests: (i) One type of post-hoc tests is the class-specific tests discussed in Dixon (2002) and Ceyhan (2009). (ii) For pairwise comparison of the interaction between classes, one can resort to two options: (a) in an $m \times m$ NNCT, one can consider cell-specific tests for each cell (which also provides interaction of the class with itself on the diagonal cells) and (b) one can restrict attention to the pair of classes i, j with $i \neq j$ one at a time and conduct the tests as in the two-class case with a 2×2 NNCT. Here the 2×2 NNCT can be formed as if only classes i and j are present in the study region ignoring the other classes. Notice that in (b), Q and R values should also be updated. We recommend the approach in (a), since it incorporates all the classes in question and provides the types of interaction in the presence of all classes, while the approach in (b) ignores the possible confounding effects of classes different from the pair in question. Furthermore, the approach in (b) might not give the exact picture of the mixed relationships between all the classes in practice.
(iii) Alternatively, for class i, we can pool the remaining classes and treat them as the other class in a two-class setting. Then we apply the two-class tests to the resulting NNCT. To emphasize the difference, this version of the class-specific test is called *one-vs-rest type test*.

4 Empirical Size and Power Analysis in the Three-Class Case

In this section, we provide the empirical significance levels for the overall and cell-specific segregation tests in the three-class case under RL and CSR independence patterns.

4.1 Empirical Size Analysis under CSR Independence of Three Classes

The symmetry in cell counts for rows in Dixon's cell-specific tests and columns in the new cell-specific tests occur only in the two-class case. To assess the performance of the cell-specific and overall tests better, we also consider the three-class case. In the three-class case, we label the classes as X, Y, and Z or classes 1, 2, and 3, interchangeably. We generate n_1, n_2, n_3 points distributed independently uniformly on the unit square $(0, 1) \times (0, 1)$ from these classes.
For the cell-specific tests, type I and III tests are closer to the desired level, and are less affected by the differences in class sizes. On the other hand, Dixon's test is extremely liberal or conservative, when class sizes are very different (which may result in smaller expected cell counts). The overall tests have similar size performance with Dixon's test being slightly better for classes of smaller size, while type I and III slightly better for classes of larger size.
The empirical size performance under RL is similar (see Ceyhan (2013)), hence is not presented.

4.2 Empirical Power Analysis under Segregation of Three Classes

Under the segregation alternatives for three classes, we generate $X_i \stackrel{iid}{\sim} \mathcal{U}(S_1)$, $Y_j \stackrel{iid}{\sim} \mathcal{U}(S_2)$, and $Z_k \stackrel{iid}{\sim} \mathcal{U}(S_3)$ for $i = 1, \ldots, n_1$, $j = 1, \ldots, n_2$, and $k = 1, \ldots, n_3$ where $S_1 = (0, 1 - 2s) \times (0, 1 - 2s)$, $S_2 = (2s, 1) \times (2s, 1)$, and $S_3 = (s, 1 - s) \times (s, 1 - s)$ with $s \in (0, 1/2)$. We consider the following segregation alternatives:

$$H_{S_1} : s = 1/12, \quad H_{S_2} : s = 1/8, \text{ and } H_{S_3} : s = 1/6. \tag{21}$$

Notice that, as s increases, segregation between the classes gets stronger; that is, segregation gets stronger from H_{S_1} to H_{S_3}. Furthermore, by construction classes X and Y are more segregated compared to Z and X

or Z and Y. In fact, the segregation between X and Z and segregation between Y and Z are identical (as a stochastic process).
For diagonal cells $(1,1)$ and $(2,2)$ type I and III tests have higher power, while for diagonal cell $(3,3)$, all tests have similar power estimates. For the off-diagonal cells $(1,2)$ and $(1,3)$ all tests have similar power estimates (although type I and III tests have slightly higher power), while for cell $(2,3)$ type I and III tests have higher power. In line with our simulation setup, power estimates for cells $(1,1)$ and $(2,2)$ are higher compared to cell $(3,3)$, as classes X and Y are more segregated compared to class Z. For the same reason, power estimates for cell $(1,2)$ is higher compared to cells $(1,3)$ and $(2,3)$. Type I and III overall tests have higher power compared to Dixon's test.

4.3 Empirical Power Analysis under Association of Three Classes

Under the association alternatives for three classes, we also consider three cases. We generate $X_i \stackrel{iid}{\sim} \mathcal{U}((0,1)\times(0,1))$ for $i=1,2,\ldots,n_1$. Then we generate Y_j and Z_k for $j=1,2,\ldots,n_2$ and $k=1,2,\ldots,n_3$ as follows. For each j, select an i randomly, and set $Y_j := X_i + R_j^Y\,(\cos T_j, \sin T_j)'$ where $R_j^Y \stackrel{iid}{\sim} \mathcal{U}(0,r_y)$ with $r_y \in (0,1)$ and $T_j \stackrel{iid}{\sim} \mathcal{U}(0,2\,\pi)$. Similarly, for each k, select an i' randomly, and set $Z_k := X_{i'} + R_k^Z\,(\cos U_\ell, \sin U_\ell)'$ where $R_k^Z \stackrel{iid}{\sim} \mathcal{U}(0,r_z)$ with $r_z \in (0,1)$ and $U_k \stackrel{iid}{\sim} \mathcal{U}(0,2\,\pi)$. We consider the following association alternatives:

$$H_{A_1}: r_y = 1/(2\sqrt{n_t}),\; r_z = 1/(3\sqrt{n_t}),\quad H_{A_2}: r_y = 1/(2\sqrt{n_t}),\; r_z = 1/(4\sqrt{n_t}),\\ \text{and } H_{A_3}: r_y = 1/(3\sqrt{n_t}),\; r_z = 1/(4\sqrt{n_t}) \quad (22)$$

where $n_t = n_1+n_2+n_3$. As r_y and r_z decrease, the level of association increases. That is, the association between X and Y and association between X and Z get stronger from H_{A_1} to H_{A_3}. By construction, classes Y and Z are associated with class X, while classes Y and Z are not associated, but perhaps mildly segregated for small r_y and r_z. Furthermore, by construction, classes X and Z are more associated compared to classes X and Y.
For cells $(1,2)$ and $(1,3)$, type I and III cell-specific tests have higher power, while for cells $(2,1)$, $(3,1)$, $(2,3)$ and $(3,2)$, Dixon's cell-specific test has higher power. For the overall tests, Dixon's test has higher power estimates.

Remark 4.1. **Empirical Size and Power Analysis for the One-vs-Rest Type Tests in the Three Class Case** In terms of empirical size, among cell-specific tests, type I and type III tests perform better compared to Dixon's test, since they are closer to the nominal level especially for large classes. For the overall tests, the tests are about the nominal level with type I and III tests being slightly closer than Dixon's test.
Under segregation, among the tests, type I and III tests have higher power estimates compared to Dixon's test. One class-vs-rest tests for classes 1 and 2 have higher power estimates compared to that of class 3. This occurs, since by construction, classes 1 and 2 are equally segregated from other classes, and these classes are more segregated compared to class 3.
Under association, for the one-vs-rest cell-specific tests, Dixon's test has higher power for class 1-vs-rest and 2-vs-rest tests, and type I and III have higher power for class 3-vs-rest test. For the overall one-vs-rest tests, Dixon's test has higher power for classes 1 and 2, and for class 3, all tests have similar power estimates. □

5 Example Data: Swamp Tree Data

The NNCT methodology is illustrated on an ecological data set: the swamp tree data of Good and Whipple (1982) which was also analyzed by Dixon (1994, 2002). Briefly, the plot contains 13 different tree species, of which four species account for over 90 % of the 734 tree stems. In our analysis, we only consider black gums (*Nyssa sylvatica*), Carolina ashes (*Fraxinus caroliniana*), and bald cypresses (*Taxodium distichum*) as if only these three tree species exist in the area, so we are ignoring the possible effects of other species on the spatial interaction between these species for illustrative purposes. Thus, we perform a 3×3 NNCT-analysis on this data set.

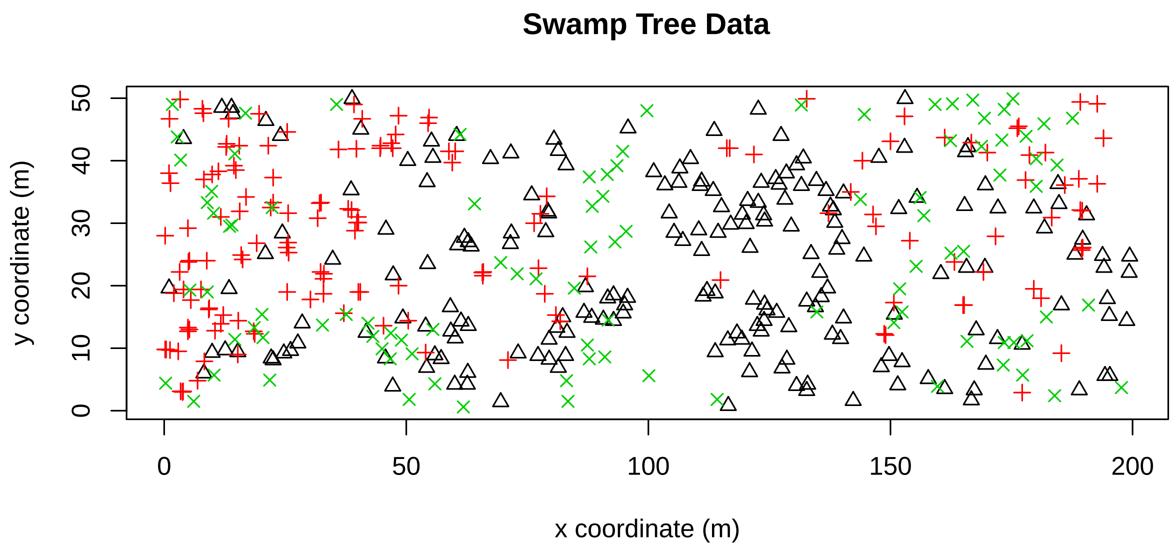

Figure 1: The scatter plot of the locations of black gum trees (triangles $\triangle$), Carolina ashes (pluses $+$), bald cypress trees (crosses $\times$).

		NN			
		B.G.	C.A.	B.C.	sum
base	B.G.	142 (69 %, .31)	40 (20 %, .09)	23 (11 %, .05)	205 (45 %)
	C.A.	34 (22 %, .07)	97 (62 %, .21)	25 (16 %, .05)	156 (34 %)
	B.C.	38 (39 %, .08)	32 (33 %, .07)	28 (29 %, .06)	98 (21 %)
	sum	214 (47 %)	169 (37 %)	76 (17 %)	459 (100 %)

Table 2: The NNCT for swamp tree data and the corresponding percentages and $\widehat{\pi}_{ij} = N_{ij}/n$ values (in parentheses), where the cell percentages are with respect to the size of the base species (i.e., row sums), and marginal percentages are with respect to the total size, n. B.G. = black gums, C.A. = Carolina ashes, and B.C. = bald cypresses.

Overall tests

	$\mathcal{X}_D$	$\mathcal{X}_I$	$\mathcal{X}_{III}$
	75.78	65.35	65.39
p_{asy}	$< .0001$	$< .0001$	$< .0001$
p_{mc}	$< .0001$	$< .0001$	$< .0001$
p_{rand}	$< .0001$	$< .0001$	$< .0001$

Table 3: Test statistics and p-values for the overall tests and the corresponding p-values. p_{asy}, p_{mc}, and p_{rand} stand for the p-values based on the asymptotic approximation, Monte Carlo simulation, and randomization of the tests, respectively. $\mathcal{X}_D$ stands for Dixon's overall test, $\mathcal{X}_I$ and $\mathcal{X}_{III}$ are for type I and type III overall tests, respectively.

Dixon's cell-specific tests

	B.G.	C.A.	B.C.
B.G.	6.57 ($< .0001$, $< .0001$, $< .0001$,)	-4.46 ($< .0001$, $< .0001$, $< .0001$,)	-3.74 (.0002, $< .0001$, .0003)
C.A.	-5.65 ($< .0001$, $< .0001$, $< .0001$)	6.60 ($< .0001$, $< .0001$, $< .0001$)	-1.70 (.0893, .0918, .1032)
B.C.	-1.18 (.2395, .2470, .2596)	-0.30 (.7672, .7796, .8140)	1.51 (.1320, .1345, .1445)

Type I cell-specific tests

	B.G.	C.A.	B.C.
B.G.	6.91 ($< .0001$, $< .0001$, $< .0001$)	-6.29 ($< .0001$, $< .0001$, $< .0001$)	-2.37 (.0177, .0170, .0176)
C.A.	-6.86 ($< .0001$, $< .0001$, $< .0001$)	6.49 ($< .0001$, $< .0001$, $< .0001$)	-0.21 (.8352, .8439, .8408)
B.C.	-1.67 (.0944, .0954, .0900)	-0.96 (.3382, .3407, .3433)	2.61 (.0091, .0087, .0081)

Type III cell-specific tests

	B.G.	C.A.	B.C.
B.G.	6.91 ($< .0001$, $< .0001$, $< .0001$)	-6.29 ($< .0001$, $< .0001$, $< .0001$)	-2.37 (.0180, .0172, .0179)
C.A.	-6.86 ($< .0001$, $< .0001$, $< .0001$)	6.49 ($< .0001$, $< .0001$, $< .0001$)	-0.20 (.8381, .8455, .8436)
B.C.	-1.67 (.0943, .0953, .0898)	-0.96 (.3375, .3401, .3426)	2.60 (.0094, .0088, .0084)

Table 4: Test statistics and p-values for the cell-specific tests and the corresponding p-values (in parentheses). The p-values are given in the order of p_{asy}, p_{mc}, and p_{rand}, whose labeling is as in Table 3. B.G. = black gums, C.A. = Carolina ashes, and B.C. = bald cypresses.

In the swamp tree data, the locations of the tree species can be viewed a priori resulting from different processes, so the more appropriate null hypothesis would be the CSR independence pattern (Goreaud and Pélissier (2003)). We compute $Q = 282$ and $R = 288$ for this data set and our inference will be conditional on these values. Dixon's and the new overall segregation tests and the associated p-values are presented in Table 3, where p_{asy} stands for the p-value based on the asymptotic approximation, p_{mc} is the p-value based on 10000 Monte Carlo replication of the CSR independence pattern in the same plot and p_{rand} is based on Monte Carlo randomization of the labels on the given locations of the trees 10000 times. Notice that p_{asy}, p_{mc}, and p_{rand} are all significant. The cell-specific test statistics and the associated p-values are presented in Table 4, where p-values are calculated as in Table 3. Again, all three p-values in Table 4 are similar for each cell-specific test.

6 Discussion and Conclusions

We survey the cell-specific and overall segregation tests based on nearest neighbor contingency tables (NNCTs) that exist in literature, introduce new ones and discuss their properties. NNCT-tests are used in testing randomness in the nearest neighbor (NN) structure between two or more classes with NN probabilities being proportional to the class frequencies. The overall test is used for testing any deviation from the null pattern in all of the NNCT cells combined; cell-specific test for cell (i,j) is used for testing any deviation from the null case in cell (i,j), i.e., the probability of a (base,NN) pair in which base class is i and NN class is j being proportional to the product of frequencies of classes i and j. This statistic tests the segregation or lack of it, if $i=j$; the association or lack of it between classes i and j, if $i \neq j$. Among many possible patterns, the null pattern is implied by the RL or CSR independence patterns. However, under the CSR independence pattern, NNCT-tests are conditional on Q and R, while under the RL pattern, these tests are unconditional. Although we consider five types of cell-specific and overall tests, we demonstrate that essentially, these tests yield three distinct types of cell-specific and overall tests. More specifically, Dixon's tests and type II tests are identical, and so are type III and type IV tests. We observe that the cell-specific tests tend to standard normal distribution, as the class sizes get larger. On the other hand, the overall tests tend to chi-square distribution with the corresponding degrees of freedom with the increasing class sizes. In terms of the asymptotic distribution of the overall tests, we have two groups of tests. For m classes, Dixon's overall test has χ^2 distribution with $m(m-1)$ df, while type I and III tests have χ^2 distribution with $(m-1)^2$ df. The results obtained from our extensive Monte Carlo simulations are summarized below:

- **Sample Size Requirement for Asymptotic Approximation:** The asymptotic approximation for the cell-specific-tests is appropriate only when the corresponding cell count in the NNCT is larger than 10; and for the overall tests when all cell counts are at least 5. For NNCTs with smaller cell counts, we recommend the Monte Carlo randomization of the tests when the null hypothesis is RL, and we recommend the use of Monte Carlo critical values when the null hypothesis is CSR independence.
- **Empirical Size Performance:**
  - In the two-class case, type I and type III cell-specific tests have better performance for the cell corresponding to the smaller class, while Dixon's cell-specific test has better performance for the cell corresponding to the larger class. For the overall test, the performance of the tests are similar for Dixon's and type I and type III tests.
  - In the three class case, type I and type III cell-specific tests have better performance, and overall tests have similar size estimates. We also observe that type I and III cell-specific tests and type III overall test are more robust to the differences in class sizes (i.e., differences in relative abundances).
- **Empirical Power Performance under Segregation:**
  - In the two-class case, type I and type III cell-specific tests have similar power estimates which are larger than those of Dixon's, and the same holds for the overall tests.
  - In the three class case, type I, III and Dixon's cell-specific tests have similar power estimates, with type I and type III being slightly higher. The same holds for the overall tests.
- **Empirical Power Performance under Association:**
  - In the two-class case, type I and type III cell-specific and overall tests tend to have higher power estimates for most of the class size combinations. The only exception is when the classes are highly unbalanced and the cell-specific test is for the diagonal cell with the larger class. In this case, Dixon's tests have higher power.
  - In the three class case, type I and type III cell-specific tests have higher power estimates for cell (i,j), if n_i is less than n_j, while Dixon's cell-specific tests have higher power estimates if n_i is larger than n_j.
  - For the overall tests, Dixon's overall test has the highest power estimates.
- **Overall Recommendations:**
  - When empirical size and power performances are considered together, among cell-specific tests, type I and type III cell-specific tests are recommended against the segregation alternatives, while type I, type III, and Dixon's cell-specific tests are recommended against the association alternatives depending on the class sizes in the off-diagonal cells.

– Among overall tests, type I and type III overall tests are recommended against the segregation alternatives, while Dixon's overall test is recommended against the association alternatives.

– We extend this recommendation to one-vs-rest type tests as well. Furthermore, for one-vs-rest type tests, all the tests have similar size performance, but type I and type III are more robust to differences in relative abundances. □

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