Functional linear models driven by point processes

José C. S. de Miranda
Institute of Mathematics and Statistics - University of São Paulo, São Paulo, Brazil - simon@ime.usp.br

Functional data analysis is now a well established field of statistics. It deals with functional variables instead of numeric or finite dimensional vector ones. A large number of univariate or multivariate statistical models have their analogs in the functional setting. For example, in functional data analysis one studies functional linear models which are the analogs of linear models in finite dimensional Statistics. When studying these models one is often interested in estimating a kernel, a function that in functional data analysis plays the role of the slope in standard linear models. In this work we will be interested in a new functional model where the kernel is now replaced by a point process. Point process may be interpreted as a random choice of a collection of points in some space. One of the most known point process is the Poisson process on the real line. The aim of this work is to introduce a new class of functional models where a point process is part of the model and plays a key role in it. This is a generalization of the classical functional model setting. Inference results for the linear case are also presented. We focus on a functional linear model where the kernel is a stochastic point process, N. If N is a Poisson process, we derive unbiased and consistent estimators of the intensity of N, in the homogeneous case, and of the expectation measure of N, in the non homogeneous case. We remind that these quantities define the law of the Poisson process. Asymptotic normality of these estimators is obtained under regularity conditions on the functional noise. From the practical point of view, these models may be used in situations where one has random shocks that interact with some functional input. This situation is common in finance and economics.

Keywords: Poisson process; Functional data analysis; Estimation of intensity; Expectation measure.