Measure of multidimensional poverty
Robustness of indices to weighting schemes

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Abstract

This paper analyzes the robustness of multidimensional poverty indexes by considering various weighting schemes. In particular, it establishes comparison between three approaches for measuring multidimensional poverty, namely: fuzzy logic method, Alkire and Foster (AF) method and Bourguignon & Chakravarty method.

By combining two approaches: A quantitative approach of factor analysis and a qualitative one taking into account the perception of deprivation by the poor, this paper develops an empirical approach to determine dimensions of multidimensional poverty. By referring to the analytical frame of capabilities as developed by A. Sen, a list of central and basic functioning was chosen.

The results show that all poverty indices estimated by using the AF weighting scheme are significantly higher than those estimated using the other pre-determined normative weights. The stochastic dominance of the curves of poverty allowed us to confirm the robustness of these results.

Moreover, if the items of deprivation are not structured by dimension, the results show, whatever selected weighting schemes, an overestimation of the indices of multidimensional poverty, in particular those obtained through the AF weighting scheme. This pattern was consistent for all weighting schemes and impacts both the headcount of poverty as well as the multidimensional poverty index (MPI).

The results of Bourguinon and Chakravarty method show that, whatever is the adopted weighting scheme, the indices of multidimensional poverty are lower than those obtained from the AF approach. The differences become more important when adopting the weighting scheme suggested by AF. Besides, the differences observed become more pronounced if we don't structure the items of deprivation by dimension. Also, the multidimensional index of poverty obtained according to fuzzy logic method not only shows an important sensibility to the weighting design but also it remains widely lower than those obtained according to the approach A.F.

Keywords: weighting schemes; multidimensional poverty; robustness of indices.

Introduction

In recent years, despite the many methods available for measuring multidimensional poverty, some important methodological problems remain unsettled. They relate mainly on four points: (i) the identification of the relevant dimensions of deprivation; (ii) the aggregation of the dimensional indications; (iii) the choice of the weighting scheme; and (iv) the determination of the poverty line by dimension. Each of these methodological items widely affects the targeting of the poor and the choice of relevant economic and social policies especially those aiming to reduce poverty and inequalities.

The main purpose of this work is to assess the robustness of the multidimensional indices poverty according to different weighting schemes. So, it is necessary to make the tour of the methodological frames, with a particular focus on the approach of the capabilities of A. Sen, which establishes an adequate frame to measure the multidimensional poverty. To implement this approach, three methodologies of measurement will be applied while adopting a range of weighting schemes: (i) the fuzzy logic method; (ii) the Bourguinon and Chakravarty method; and (iii) the AF method.

I. Weighting schemes: presentation of the main statistical methods

Every measure of multidimensional poverty sets somehow a weight to each well-being dimension. However, the weighting scheme can vary in its specification and the way it affects the estimation of

weighted indices. From then on, the robustness of the multidimensional indices to a range of weighting schemes continues to be a serious challenge.

In this regard, it is important to analyze the sensibility of the poverty indices according to various weighting schemes. Since 1988, different statistical weighting schemes have been designed to facilitate the summing of dimensional indices in a composite index. Generally, three statistical functions of weighting can be distinguished, such as: The specifications of Desai & Shah (1988), Cerioli & Zani (1990) and Betti & Verma (1998) and Betti & al (2007).

Although there are several possible formulations of these types of functions, we present below some that are usually used to determine the weighting coefficients :

i) The function of normalized weighting proposed by Cerioli and Zani (1990):

$\mathbf{w_{CZ}^{j}} = \frac{\mathbf{ln}(\sum_i \mathbf{f(a_i)}/(\sum_i \mathbf{f(a_i)x_{ij}}))}{\sum_j \mathbf{ln}(\sum_i \mathbf{f(a_i)}/(\sum_i \mathbf{f(a_i)x_{ij}}))}$ with f ($\mathbf{a_i}$) is the weight attached to the observation of the sample $\mathbf{a_i}$, $x_{ij} \in [0, 1]$ denotes the value of a particular deprivation item j.

This formulation shows that the weight attributed to the factor j is an inverse function of its degree of deprivation.

Ceriolis and Zani also developed another not logarithmic format: $\boldsymbol{w_{CZ-alt}^{j}} = \frac{(\sum_i \boldsymbol{f(a_i)}/(\sum_i \boldsymbol{f(a_i)x_{ij}})}{\sum_j(\sum_i \boldsymbol{f(a_i)}/(\sum_i \boldsymbol{f(a_i)x_{ij}}))}$

ii) The function of normalized weighting proposed by Desai and Shah (on 1988)

$$\boldsymbol{w_{DS}^{j}} = \frac{\mathbf{1} - \left((\sum_i \boldsymbol{f(a_i)x_{ij}})/\sum_i \boldsymbol{f(a_i)}\right)}{\sum_j \left(\mathbf{1} - \left((\sum_i \boldsymbol{f(a_i)x_{ij}})/\sum_i \boldsymbol{f(a_i)}\right)\right)}$$

In this specification, although the weighting function grants more weight for the deprivation, the fact of not considering a logarithmic function allows to grant more importance for indices of deprivation translating less frequent symptoms of poverty. This approach tends to converge the weights of items measuring the deprivation. Unlike the approach of Cerioli and Zani which overrepresented the weight of the least wide-spread deprivation.

iii) weighting scheme of Betti & Verma

This weighting scheme is based on two principles: i) the weight to be attributed measures the intensity of deprivation of an attribute; its value is an inverse function of the degree of deprivation of this attribute for the population; ii) the index measuring this weight aims to reduce the over-representation due to the risks of the high correlation between the attributes and to the redundancy of the information. So, the suggested function removes from the calculation items bringing the same information by eliminating their weights.

$\boldsymbol{w_{BV}^{j}} = \frac{\boldsymbol{w_a^j} \times \boldsymbol{w_b^j}}{\sum_{m=1}^{M} \boldsymbol{w^m}}$ with $\mathbf{w_a^j} = \frac{(\sum_i \mathbf{f(a_i)}\ (\mathbf{x_{ij}} - \bar{\mathbf{x}}_\mathbf{j})^2)^{\frac{1}{2}}}{(\sum_i \mathbf{f(a_i)})^{\frac{1}{2}} \bar{\mathbf{x}}_\mathbf{j}}$ and

$$\boldsymbol{w_b^j} = \left(\mathbf{1} + \sum_{m=1}^{M} \boldsymbol{\rho_{jm}} . \mathbf{I}(\boldsymbol{\rho_{jm}} < \boldsymbol{\rho_h})\right)^{-1} \times \left(\sum_{m=1}^{M} \boldsymbol{\rho_{jm}} . \mathbf{I}(\boldsymbol{\rho_{jm}} \geq \boldsymbol{\rho_h})\right)^{-1}$$

$\boldsymbol{w_a^j}$ depends on the distribution of the attribute j in the population, and $\boldsymbol{w_h^j}$ depends on the correlation between x_i and other attributes, it measures the average correlation of item j with all the other items. The more it increased the lower is the weight of the attribute j. $\boldsymbol{\rho_{jm}}$ represents the level of correlation between two attributes j & m, I(.) is an indicator function and $\boldsymbol{\rho_h}$ is a pre-determined cutoff correlation level (in our case $\rho_h = 0.5$).

iv) The third weighting scheme combines the advantages of Cerioli-Zani method and Betti-Verma method : It attributes an upper weight for the least wide-spread deprivation, and limites the influence of the correlation and the redundancy of the information on the weighting. Within the framework of this study, we adopted the following both specifications:

$$W^{j}_{BV-CZ} = \frac{(\ln(\sum_i f(a_i)/(\sum_i f(a_i)x_{ij})))}{\left(1+\sum_{m=1}^{M}\rho_{jm}\left|\, I(\rho_{jm}<\rho_h)\right.\right)\left(\sum_{m=1}^{M}\rho_{jm}\left|\, I(\rho_{jm}\geq\rho_h)\right.\right)}$$

This function would allow to attribute a weight to the item j by combining the methods of Betti&Verma and Cerioli & Zani.

Similar to the specification BV-CZ, the second specification would allow us to adjust the function of the weight proposed by Desai-Shah by introducing the measure of the average correlation which grants less weight for the strongly correlated attributes.

$$W^{j}_{BV-DS} = \frac{(1-(\sum_i f(a_i)x_{ij})/\sum_i f(a_i))}{\left(1+\sum_{m=1}^{M}\rho_{jm}\left|\, I(\rho_{jm}<\rho_h)\right.\right)\left(\sum_{m=1}^{M}\rho_{jm}\left|\, I(\rho_{jm}\geq\rho_h)\right.\right)}$$

v) Linear weighting scheme of A.F: this weighting scheme gives equal weighting to each dimension and each item in every dimension.

$W^{j}_{A.F} = \frac{1}{D.K_j}$ **w**ith D: number of dimensions and K the number of items in the dimension containing the item j.

Definition of the space of deprivation : central and basic functioning and indices of measure

Dimension	**central and basic functioning**	**indicators of measure**
Education	Schooling for adults	The number of members of the household having completed 5 years of schooling.
	Schooling of children	Number of chidren of school age attending the school.
Maternal health and children's nutrition	medicale consultation	Sick individuals who have access to a medical consultation.
	Deliveries under medical supervision	Deliveries occured in watched environment
	Children nutrition	Number of undernourished children
Economic power	Income inequality	Income per capita
Labour market participation	To be put in job	Number of unemployed by household
Food consumption	Provide red or white meats and fish	per capita average spending for meats and fishes
Housing conditions	Sanitary equipments	Sanitary equipments (toilet, washbasin, bath/shower, sewers)
	Domestic equipments	Domestic equipments (Tv, radio, telephone, refrigerator, bicycle, moped, car, tractor)
	Electricity	Access to electricity
	Water	Access to drinking water

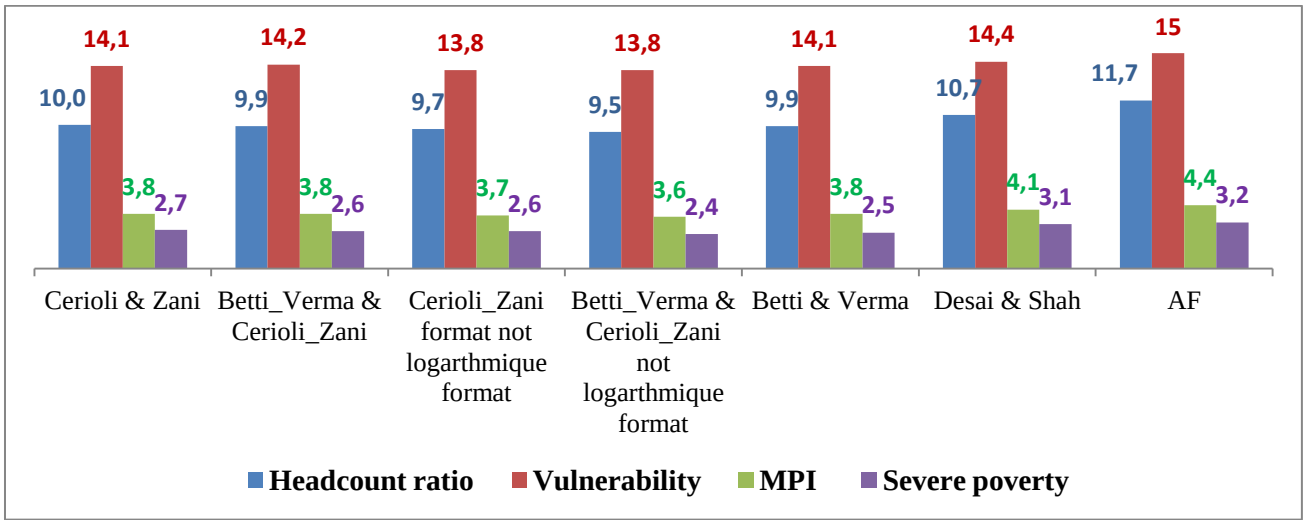
Source : the national survey on living conditions, 2007, HCP, Morocco

II. Summary of the results : measurement indices of poverty according to approaches of measure and weighting schemes

By adopting AF method, the results show that all indices of poverty (headcount ratio of poverty,

multidimensional poverty index (MPI), vulnerability and severe poverty) estimated according the AF weighting scheme are significantly higher than those estimated using the other pre-determined normative weights. The stochastic dominance of the curves of poverty allows us to confirm the robustness of these results.

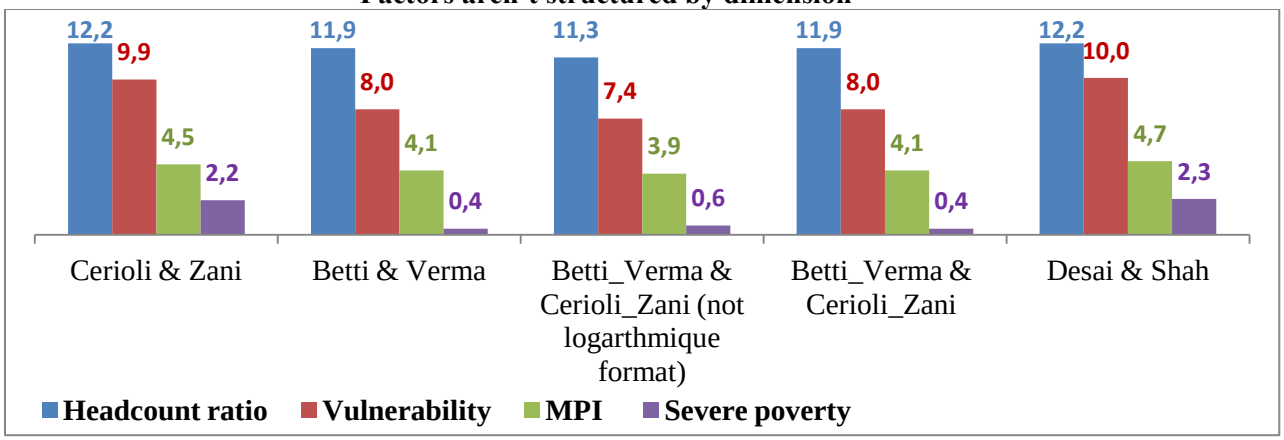
Graph 1 : Multidimensional indices of poverty according to AF method : sensibility to weighting schemes. Factors structured by dimension (%)



Source: Author's calculation

If we don't structure the items by dimension we'll have an overvaluation of the index of multidimensional poverty, in particular when using the AF weighting scheme. This tendency concerns all the weighting schemes, the headcount of poverty and the multidimensional poverty index (MPI). In light of these results, it turns out important to rethink the broad configuration of the deprivation items by dimension.

Graph 2 : Multidimensional indices of poverty according to the approach AF Factors aren't structured by dimension

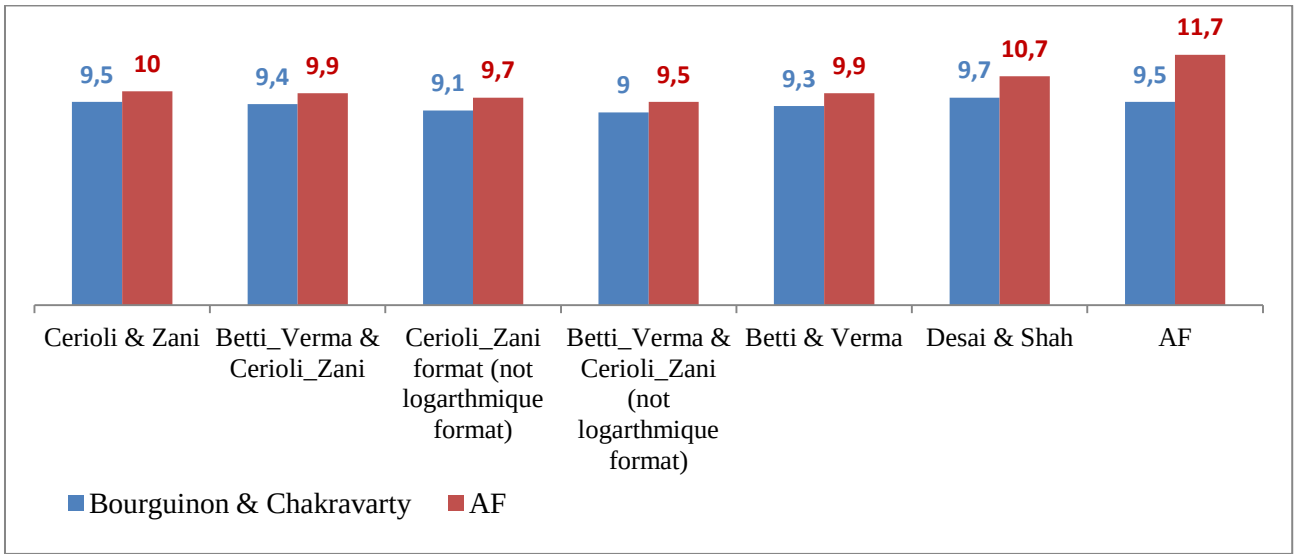


Source: Author's calculation

By structuring the items of deprivation by dimension, the results of the fuzzy logic approach show that, regardless of weighting scheme, the indices of poverty are lower than those emanating from the AF approach. When the items of deprivation are not structured by dimension the observed differences become more significant. Besides, these differences also show the tendency pronounced by the approach AF to overestimate the incidence of poverty in comparison with the fuzzy logic approach. Straightaway, by referring to the approach of Bourguinon and Chakravarty (2002), the

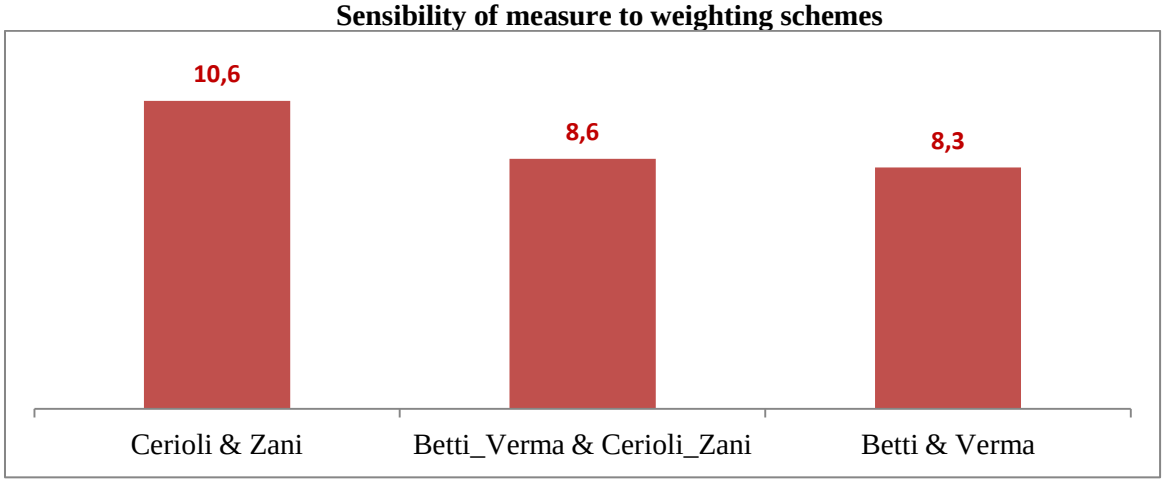
multidimensional indices of poverty obtained while showing a sensibility important for the weighting schemes, they remain lower than those obtained according to the AF method.

Graph 3 : Comparison of headcount ratio between AF method and Bourguinon & Chakravarty method. Factors structured by dimension (%)



Source: Author's calculation

Graph 4: Headcount ratio of poverty[1] according to fuzzy logic method
Sensibility of measure to weighting schemes



Source: Author's calculation

Conclusion

This paper studied the sensitivity of multidimensional indices of poverty to different weighting schemes. Our approach explained why multidimensional poverty measure depends not only on the approach but also on the weighting scheme adopted. Testing of pre-determined normative weighting schemes showed that the linear weighting overestimates poverty indices. Whatever the weighting scheme adopted, an important difference is observed in the estimation of poverty indices depending on whether the items of deprivation are organized or not by dimension. The most important difference

[1] According to fuzzy logic method, a household is considered poor if it accumulates at least 30 % of the deprivations among the dimensions determining the well-being.

concerns the AF approach. This tendency concerns all the multidimensional indices of poverty. The stochastic dominance of the curves of poverty allowed us to confirm the robustness of these results.

The results of Bourguinon and Chakravarty method show that, regardless of the weighting scheme, the indices of multidimensional poverty are lower than those obtained from the AF approach. The differences become more significant by adopting the weighting scheme proposed by AF. Besides, the differences noticed become more important if we don't structure the items of deprivation by dimension. Also, the multidimensional indices of poverty obtained according to fuzzy logic method not only shows an important sensibility for the weighting schemes but remains widely lower than those obtained using the A.F. approach.

In addition, this work shows that a better targeting of the poverty is not only conditioned by a determination of the relevant dimensions of the poverty but also by an adequate choice of the weighting scheme and the measurement approach.

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