RIO DE JANEIRO
iSi 2015
60th World Statistics Congress

Bayesian matrix variate elliptical model analysis

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Abstract

The matrix variate elliptical model is considered, where under a subjective Bayes viewpoint, estimators for the location matrix and the determinant of the characteristic matrix are exactly derived assuming two loss functions, with the normal-inverse Wishart and normal-Wishart, respectively, as priors. In addition, the newly developed results will be applied to the matrix variate normal distribution and the matrix variate t-distribution as particular subfamilies of the matrix variate elliptical model. A simulation study as well as a real data set are used to illustrate these new results and the usefulness of the normal-Wishart prior.

Keywords: Normal-inverse Wishart; normal-Wishart; squared error loss; maximum posterior mode.

1. Introduction

The objective of this paper is to perform a subjective Bayesian analysis for the matrix variate elliptical model with density function given by

$$f(\mathbf{X}) = c_{s,p}|\mathbf{\Sigma}|^{-\frac{p}{2}}|\mathbf{\Omega}|^{-\frac{s}{2}}h[tr(\mathbf{X}-\boldsymbol{\mu})'\mathbf{\Sigma}^{-1}(\mathbf{X}-\boldsymbol{\mu})\mathbf{\Omega}^{-1}]. \quad (1)$$

The location and scale matrices are respectively assumed to be $\boldsymbol{\mu}_{p\times s}$ and $\mathbf{\Sigma}_{p\times p}\otimes\mathbf{\Omega}_{s\times s}$ where $\otimes$ denotes the Kronecker product. The considered prior distributions will thus be for $\boldsymbol{\mu}_{p\times s}$ and $\mathbf{\Sigma}_{p\times p}$. (See Fang and Zhang (1990)) Subjective analysis generally produces more admissible results compared to objective analysis, since added information is used. Although the latter was considered by Fang and Li (1999), very few results and estimators for the Bayesian analysis of the matrix variate elliptical model exist (see Arashi et.al. (2013) for a conjugate prior). This paper contributes to the literature by presenting a more general subjective Bayesian estimation framework for the matrix case, see (1). Prior information will be reflected by using the normal-inverse Wishart and the normal-Wishart prior distributions respectively (see Van Niekerk et.al. (2013) for the multivariate elliptical model and Bekker and Roux (1995) for the normal model)

The squared error loss function as well as the loss function defined by Das and Dey (2010) will be used for Bayesian inference. The paper is structured as follows: In section 2, results for the posterior distributions, Bayes estimators of the location matrix $\boldsymbol{\mu}$, and the determinant of the characteristic matrix $\mathbf{\Sigma}$ of the matrix variate elliptical model for the normal-inverse Wishart prior and the normal-Wishart prior are given. The newly developed results of section 2 will subsequently be applied to particular subfamilies of the matrix variate elliptical distribution in section 3.

2. Statistical properties

The exact posterior densities for the parameters are derived as well as some statistical properties like the joint density function of the eigenvalues of the characteristic matrix using the posterior distribution. The Bayesian estimators are derived in an explicit form using two different loss functions.

3. Application

The results derived in section 2 are illustrated using a simulation study and a real dataset. Gibbs sampling was used. It was found that the normal-Wishart prior produced a less bias estimate than the objective prior as well as the normal-inverse Wishart prior.

4. Conclusions

In this paper, the matrix variate elliptical model was observed from a subjective Bayesian viewpoint. The normal-inverse Wishart and normal-Wishart priors were considered for the location and scale matrices of the underlying model. In summary, the joint posterior density functions, marginal posterior density functions and Bayes estimators, under two loss functions, of the parameters were found, with the matrix variate normal and matrix variate t-distribution as particular subfamilies. The Bayesian estimator under SEL of the location parameter is a robust estimator in the sense that it is independent of the prior distribution of the scale parameter. For both priors, the posterior distributions for the location and scale matrices of the matrix variate normal model were simulated, the Bayes estimators under the two loss functions were calculated, and the cumulative density function of the largest eigenvalue of Σ was obtained. The simulation study justified the use of a normal-Wishart prior as a subjective prior in the Bayesian analysis of the normal model.

Acknowledgments: This work is based on research supported by the National Research Foundation. South Africa (Grant: COMPETITIVE PROGRAMME FOR RATED RESEARCHERS (CPRR 13090132066) and the University of Pretoria Doctoral research support bursary.

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