



## Expected Shortfall of Quadratic Portfolios in Non-normal Distributions

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The heavy-tailedness and the asymmetry of assets returns motivated the use of non-normal distributions for modelling risks. In [2] the hyperbolic distribution is proposed for risk modelling. In this paper, we adopt the approach of modelling the assets returns with a more general family of parametric distributions, calculating an analytical expression for the expected shortfall of quadratic portfolios when the assets returns behave by the family of the multivariate generalised hyperbolic distributions (MGH). We calculate an analytic expression of the moment generating function of the elliptical truncated first- and second-order moments of the Student's  $t$  and hyperbolic quadratic loss distribution, and by consequence we derive an analytical expression of the expected shortfall. The expected shortfall value is provided by an analytical expression. As a result, analytic expressions for the expected shortfall in the cases of normal and Student's distributions are provided, complementing the results of [1]<sup>1</sup> on heavy-tailed distributions. Resulting formulae were used to price a portfolio of options where the performance of the calculations improved the performance of the calculations using a traditional Monte Carlo methodology, and provided an analytic expression for the calculation of risk measures as the Greeks.

**Keywords:** Expected shortfall; multivariate generalised hyperbolic distribution; higher-order moments; multi-asset option pricing.

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<sup>1</sup>[1] is an extension of results of [3], from using the  $VaR$ , to using the expected shortfall as a risk measure.

## References

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