The Maximum Number of 3- and 4-Cliques within a Planar Maximally Filtered Graph

Jenna Birch*
University of Liverpool, Liverpool, UK – j.l.birch@student.liv.ac.uk

Athanasios A Pantelous
University of Liverpool, Liverpool, UK – aap@liv.ac.uk

Konstantin Zuev
University of Liverpool, Liverpool, UK – zuev@liv.ac.uk

In recent years there has been increasing interest in how we can model complex systems using network theory. In particular a network based approach of studying complex systems has become very popular in econophysics. One of the important and fundamental problems in this approach is to filter the most relevant information from financial networks. As a result, traditional algorithms from network theory have been adapted and some new methods have been introduced. One such method is the Planar Maximally Filtered Graph (PMFG) which reduces the complete network to a subgraph containing only the most relevant edges. One of the main characteristics of a PMFG is its 3- and 4-cliques which are used to analyse the network. Recently in a few high impact papers it was stated that, based on heuristic evidence, the maximum number of these 3- and 4-cliques that can exist in a PMFG with n vertices is $3n-8$ and $n-4$ respectively. In this talk we discuss the construction of the PMFG and its 3- and 4- cliques focusing on the various structures that can form including planar representations and the standard spherical triangulation. We address how the structure of the PMFG can affect the number of cliques that form within the graph. Finally we shall prove, using a set of operations proposed by Eberhard to generate planar graphs and also standard spherical triangulation that it is indeed the case that the maximum number of 3- and 4-cliques that can exist in a PMFG with n vertices is $3n-8$ and $n-4$ respectively.

Keywords: Correlation based networks; Eberhard's operation; Standard spherical triangulation.