Average derivative projection pursuit regression

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In this talk one proposes a general framework for Projection Pursuit Regression (PPR) that can deal with an explanatory random function $X = \{X(t);\ t \in \mathcal{I}\}$ and a scalar response Y. The functional PPR (see for instance Chen *et al.*, 2011) assumes that there exists an unknown orthonormal sequence of κ functional directions $\theta_1(\cdot), \ldots, \theta_\kappa(\cdot)$ such that

$$Y = \mu + m_1\left(\int \theta_1(t)X(t)dt\right) + \cdots + m_\kappa\left(\int \theta_\kappa(t)X(t)dt\right) + error,$$

where $m_1(\cdot), \ldots, m_\kappa(\cdot)$ are κ unknown univariate functions belonging to some space of smooth functions. In doing so we introduce what we call the *average derivative operator* of the regression. We show that the unknown directions involved in the functional PPR can be viewed as the eigenfunctions of the average derivative operator. This new characterization of the directional parameters greatly simplifies the estimation problem, offers new interpretation of the directional parameters in PPR and solves identifiability issues inherent in such models.

Using the work of Hall *et al.* (2009) focusing on the nonparametric estimation of the directional derivatives of a regresion operator, a nonparametric estimator of the average derivative operator is constructed. Then the above characterization of the κ directional parameters $\theta_1, \ldots, \theta_\kappa$ is used to simultaneously estimate all of them. Asymptotic properties of the introduced estimators are studied and numerical examples are used to illustrate the method and to assess finite sample performance.

Keywords: average derivative operator; functional data; projection pursuit regression.

References

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