Asymptotic Normality in Estimation of Large Ising Graphical Model

Zhao Ren*
University of Pittsburgh, Pittsburgh, USA zren@pitt.edu

Cun-Hui Zhang
Rutgers University, Piscataway, USA cunhui@stat.rutgers.edu
Harrison Zhou
Yale University, New Haven, USA huibin.zhou@yale.edu

The high dimensional graphical model, a powerful tool for studying conditional dependency relationship of random variables, has attracted great attention in recent years. This paper investigates statistical inference of each edge for some families of large graphical models which include Ising graphical model and Gaussian graphical model as two special cases. Unlike the Gaussian graphical model, in general there is no explicit correspondence between the structure of the graph and precision matrix of the underlying data. Hence this inference problem is different from inference of precision matrix and is very challenging. In this paper, we propose a novel estimator of each edge and show that, under certain sparsity assumption, our estimator is asymptotically normal and has parametric square-root rate in a large graphical model. Our proof applies a linearization idea and a novel projection procedure which is motivated by statistical inference in high dimensional regression. A careful analysis of this new methodology relaxes the commonly imposed sparsity assumption, uniform signal strength condition, bounded maximum neighborhood weight and incoherence condition in the literature of large Ising graphical model.

Keywords: Ising Graphical Model; Asymptotic Normality; Statistical Inference; Generalized Linear Models.