Asymptotic confidence sets
for parameters in high-dimensional models

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We consider the problem of constructing asymptotic confidence sets when the model is high-dimensional but sparse. We study for example the linear model

$$Y = X\beta^0 + \epsilon,$$

where Y is an n-vector of observations, X is an $n \times p$ matrix, β^0 is a p-vector of unknown coefficients and ϵ is an unobserved noise vector. The number of parameters p is possibly larger than the number of observations n. Our aim is to construct asymptotic confidence sets for a sub-vector $\beta_J^0 := \{\beta_j^0 : \ j \in J\}$ with $J \subset \{1, \ldots, p\}$ a given subset of indices. To this end, we consider as in [Zhang and Zhang(2014)] a one-step estimator

$$\hat{b}_J := \hat{\beta}_J + \hat{\Theta}_J^T X^T \hat{\epsilon}/n,$$

where $\hat{\beta}$ is an initial estimator, $\hat{\epsilon} := Y - X\hat{\beta}$ and $\hat{\Theta}_J$ is an estimate of the inverse Fisher-information. The goal is to show that $\hat{b}_J$ is asymptotically linear and to estimate its asymptotic covariance matrix. We study as initial estimator

$$\hat{\beta} := \arg\min_{\beta \in \mathbf{R}^p} \left\{ \|Y - X\beta\|_2 + \lambda\Omega(\beta) \right\},$$

with Ω a sparsity inducing norm on $\mathbf{R}^p$. Special cases are for example the square root Lasso and the square root group Lasso. For $\hat{\Theta}_J$ we also discuss various approaches based on sparsity inducing norms and we study the relation with the initial estimator. Confidence intervals for more general models will be discussed as well.

Keywords: confidence sets, high-dimensional model, sparsity.

References

[Zhang and Zhang(2014)] C.-H. Zhang and S.S. Zhang. Confidence intervals for low-dimensional parameters in high-dimensional linear models. *Journal of the Royal Statistical Society Series B*, 76:217–242, 2014.