Estimation of non-negative surfaces over complex domains using bivariate splines

Serge Guillas*
University College London, UK – s.guillas@ucl.ac.uk

Ming-Jun Lai
University of Georgia, USA – mjlai@math.uga.edu

Jean Gaudart
Aix-Marseille University, France – jean.gaudart@univ-amu.fr

We consider the estimation of a non-negative surface over a complex domain with an irregular boundary and interior holes from irregularly spaced observations. We employ bivariate splines over a triangulation to build fitting surfaces. The non-negativity condition can be satisfied by recasting the estimation of the spline's coefficients as a constrained minimization problem over the spline basis. We first show that the constrained minimization has a unique solution in the Sobolev space of twice differentiable functions over the domain. Similarly, in the finite dimensional spline spaces, there is a unique spline minimizer. We then show that the spline minimizer approximates the Sobolev minimizer as the size of the triangulation goes to zero. We finally apply a classic projected gradient method to numerically solve the minimization problem in the spline setting and show convergence. In addition, boundary conditions are enforced in accordance with the given application.
In the absence of replicates of a given surface, we quantify uncertainties through a bootstrap approach. We provide several numerical simulations to examine the merits of our approach. When a surface has less variations, the spline surface obtained by our method can fit it well. When a surface has more variations, one needs more and well distributed observations to be able to approximate the exact function well enough. An illustration to the estimation of population density of a town separated by a river highlights the skills.

Keywords: functional data; bivariate splines; spatial statistics; surface fitting.