A Markov Chain Monte Carlo Approach to Empirical Bayes Inference and Bayesian Sensitivity Analysis via Empirical Processes

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Consider a Bayesian situation in which we observe Y ∼ pθ, where θ ∈ Θ, and we have a family {νh, h ∈ H} of potential prior distributions on Θ. Let νh,y be the posterior distribution of θ given Y = y when the prior is νh, let g be a real-valued function of θ, and let Ig(h) be the posterior expectation of g(θ) when the prior is νh. We are interested in two problems: (i) selecting a particular value of h, and (ii) estimating the family of posterior expectations { Ig(h), h ∈ H . Let my(h) be the marginal likelihood of the hyperparameter h: my(h) = R pθ(y) νh(dθ). The empirical Bayes estimate of h is, by definition, the value of h that maximizes my(h). It turns out that it is typically possible to form point estimates and confidence intervals for my(h) and Ig(h) for each individual h, using Markov chain Monte Carlo. However, we are interested in estimating the entire families of integrals { my(h), h ∈ H and { Ig(h), h ∈ H : we need to estimate the first family in order to carry out empirical Bayes inference, and we need to estimate the second family in order to do Bayesian sensitivity analysis. We establish strong consistency and functional central limit theorems for estimates of these families by using tools from the theory of empirical processes. As an application, we consider the robit model in binary regression, an extension of the probit regression model, in which the normal distribution is replaced by a t distribution with d degrees of freedom, with d determining the extent of the robustness of the model against outliers. We show how our methodology can be used for making inference about d, and give an illustration on a real data set.

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