Realized Volatility of Large Portfolios

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Using high-frequency data, we estimate the risk of a large portfolio with unknown weights subject to some linear inequality constraints or known weights. We propose a fully nonparametric approach as a benchmark, as well as a factor-based semiparametric approach with observable factors to attack the curse of dimensionality. We provide in-fill asymptotic distributions of the realized volatility estimators of the optimal portfolio, while taking into account the estimation error of the optimal portfolio weights as a result of the input covariance matrix estimation. Our theoretical findings suggest that ignoring such an error leads to a first-order asymptotic bias which undermines the statistical inference. We also show that even with known portfolio weights, the semiparametric estimator suffers from a similar bias. Our simulation results suggest satisfactory finite sample performance after bias correction, and that the factor-based approach becomes increasingly superior with a growing cross-sectional dimension. Empirically, using a large cross-section of high-frequency stock returns, we find it necessary to recognize the sensitivity of portfolio risk to the statistical uncertainty of the covariance estimates.

Keywords: quadratic programing; in-sample optimism; exposure constraint; big data.