We consider the issue of bootstrapping multivariate time series. We first discuss some problems associated with applying bootstrap procedures developed for univariate time series in the multivariate context. We discuss the reasons for the failure of such time or frequency domain bootstrap methods. We then consider multivariate linear processes and show that for any bootstrap procedure to succeed it is essential that this procedure will imitate correctly also the fourth order dependence structure of the underlying process. We develop a frequency domain bootstrap procedure for the periodogram matrix and show that this procedure mimics correctly the first and the second order structure of the periodogram matrix of the observed multivariate time series including the weak dependence structure of the periodogram at different frequencies. We prove validity of our bootstrap procedure for some important classes of statistics that are commonly used in multivariate time series analysis including estimators of autocorrelation or autocovariance matrices. Some simulations illustrate our theoretical findings.

**Keywords:** Bootstrap; multivariate time series; integrated periodogram statistics; fourth order cumulants.