What makes a neuron spike - The Stochastic Hodgkin-Huxley process, periodic Harris recurrence and first steps towards statistical analysis

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The deterministic Hodgkin-Huxley model for the membrane potential of a single neuron describes the mechanism of spike generation (spikes=emission of action potentials) in response to an external input. We study a stochastic version of this model in which a cortical neuron receives some T−periodic (unknown) signal S from its dendritic system. In this frame, the stochastic Hodgkin-Huxley model is a coupled system of diffusion equations describing the observed membrane potential process (first coordinate) as well as unobserved coordinates which model ion currents. Observing the first coordinate alone leads to a non-Markovian process.
The main interest of modern neurosciences is to understand how neurons respond to external stimuli. Therefore, it is important to build statistical procedures aiming at estimating the unknown signal S (or some important features of S, for instance the unknown periodicity T), based on the observation of the membrane potential process.
In our work we establish “periodic ergodicity” of the process, based on a detailed study of the transition densities of the stochastic Hodgkin-Huxley model. The main difficulty comes from the fact that the model is a highly degenerate diffusion with time inhomogeneous coefficients. Moreover we obtain limit theorems for the sequence of successive spike intervals which allow to describe the spiking activity in the long run.

Keywords: degenerate diffusion process; weak Hoermander condition; Hodgkin-Huxley, periodic ergodicity.