

A New Notion of Depth and Central Regions for Functional Data
Naveen Naidu Narisetty
University of Michigan

There has been extensive work on data depths and their applications in the context of multivariate data. However, depth notions for infinite-dimensional objects such as functional data have received less attention. We propose a new notion of depth called Extremal Depth (ED) for functional data, discuss its properties, and compare its performance with existing concepts such as integrated data depth and band depth. The proposed notion is based on a measure of extreme 'outlyingness', similar to that for projection depth in the multivariate case. ED has many desirable properties as a measure of depth and is well suited for obtaining central regions of functional data and corresponding regions for distributions on function spaces. For constructing central regions, ED satisfies two important properties that are not shared by other notions: a) the central region achieves the nominal (desired) simultaneous coverage probability; and b) the width of the simultaneous region is proportional to that of the pointwise central regions. The empirical performance of the method is examined through simulation studies, and its usefulness is demonstrated on two applications: constructing functional boxplots and obtaining simultaneous confidence bands in parametric regression.