



## Fixed Points of the EM Algorithm and Nonnegative Rank Boundaries

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We study fixed points of the expectation-maximization (EM) algorithm and boundaries of the  $r$ -th mixture model of two discrete random variables, i.e. matrices of nonnegative rank at most  $r$ . Both problems are important to get a better understanding of maximum likelihood estimation on the mixture model. Our methods use algebraic geometry. For  $r=3$ , we give a quantifier-free formula of this semialgebraic set and describe its boundary using nested polygons. We also characterize minimal primes of the ideal of the EM fixed points and recognize the boundary components among them.

**Keywords:** maximum likelihood; mixture model.