A functional limit theorem for the partial maxima of a long memory stable sequence produces a limiting process that can be described as a beta-power time change in the classical Frechet extremal process, for beta in a subinterval of the unit interval. Any such power time change in the extremal process for $\beta < 1$ produces a process with stationary max-increments. This deceptively simple time change hides the much more delicate structure of the resulting process as a self-affine random sup measure. We uncover this structure and show that in a certain range of the parameters this random measure arises as a limit of the partial maxima of the same long memory stable sequence, but in a different space. These results open a way to construct a whole new class of self-similar Frechet processes with stationary max-increments.

**Keywords:** self-similar, heavy tails.