Star-shaped distributions: Euclidean and non-Euclidean representations

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It is well known that a spherically distributed random vector X allows the stochastic representation

$$X \stackrel{d}{=} R \cdot U \tag{1}$$

with a non-negativ random variable R being independent of a random vector U which follows the uniform distribution on the Euclidean unit sphere S. If X has a density f with density generating function g, $f(x) = C(g)g(||x||)$, where $||.||$ denotes the Euclidean norm, then the density level sets are spheres $S(r) = rS$ of positive radius r, and the distribution Φ_g of X allows the geometric measure representation

$$\Phi_g(A) = C(g) \int_0^\infty g(r) O(A \cap S(r)) dr \tag{2}$$

where O denotes the Euclidean surface content measure. Numerous probabilistic and statistical applications of this representation are surveyed in [5] and the literature mentioned there.

In [1,2,3] and in other papers the authors reveal that star-shaped sets and, correspondingly, star-shaped distributions occur in different applied disciplines.

If the density level sets of X are the boundaries $\tilde{S}(r) = r\tilde{S}$ of star bodies $\tilde{K}(r) = r\tilde{K}, r > 0$, $\tilde{S}$ different from the Euclidean sphere S, then, according to [5], O has to be replaced in (2) by a suitably defined non-Euclidean surface content measure $\tilde{O}$, and the uniform distribution of U in (1) is defined then with respect to this non-Euclidean surface measure on $\tilde{S}$.

The main aim of this talk is to discuss the intrinsic character of these non-Euclidean representations. Statistical consequences will be outlined, and special attention will be devoted to the particular case of elliptically contoured distributions which allow both matrix-transformed Euclidean and non-Euclidean representations.

The Minkowski functional of the unit ball of the chosen non-Euclidean geometry may, in dependence of the properties of $\tilde{K}$, be a norm, an antinorm, a semi-antinorm or a homogeneous functional of a type possibly not yet mentioned. For antinorms and semi-antinorms, we refer to [4].

Keywords: p-generalized elliptically contoured distributions; star-shaped sample clouds; star-shaped density level sets; generalized metric geometries.

References

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